How to detect fluctuating order in high T_c superconductors

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In memory of Victor J. Emery

Outline

1. What is "fluctuating order"

2. Quantum critical points and fluctuating order

3. Disorder as a tool to observe fluctuating order

4. Examples of fluctuating order in

   - 1DEG at $T = 0$

   - Weakly interacting 2DEG

5. Detection of fluctuating smectic (or stripe) and nematic order in High T_c superconductors: Neutron scattering, STM, and other probes.

Ordered states are characterized by a spontaneously broken symmetry $\Rightarrow$ Order Parameter

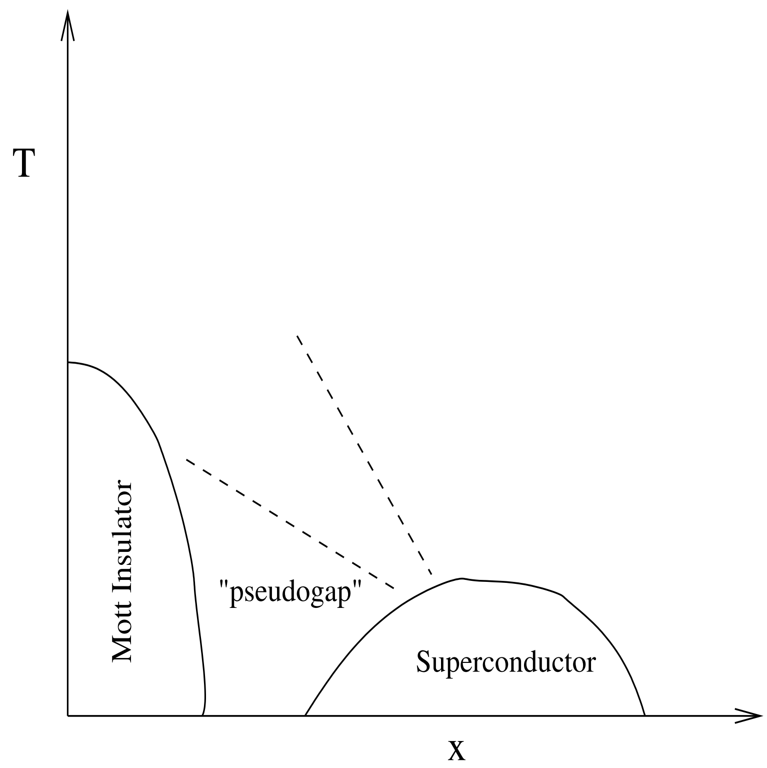
Order Parameter fluctuations grow as a (classical or quantum) critical point is approached $\Rightarrow$ Fluctuations are evidence for the proximate ordered state

Quantum Disordered Phase:
$\tau \sim E_G^{-1}$ ($E_G \equiv$ Gap) $\Rightarrow$ unless $E_G \to 0$, τ is "short"

"Fluctuating Order" is an ill-defined concept **unless** the nearby ordered state is found

Detecting Ordered States: Best way
1) detect the broken symmetry
2) detect fluctuations, e. g. measure $S(\vec{k}, \omega)$

Phase Diagram of the High T_c Superconductors



Phase diagram suggests the existence of competing phases.
Evidence for stripe charge order in underdoped HTS (LSCO and YBCO) neutrons (Tranquada), transport (Ando)
Evidence of coexistence of stripe charge order and superconductivity in LSCO and YBCO (Mook, Tranquada)
STM suggests broken rotational symmetry and short range charge order in BSCCO

Electron Liquid Crystal Phases

S. Kivelson, E. Fradkin, V. Emery, Nature **393**, 550 (1998)

Doping a Mott insulator leads to inhomogeneous phases due to the competition between phase separation and strong correlations

- Crystal Phases: break all continuous translation symmetries and rotations
- Smectic (Stripe) phases: break one translation symmetry and rotations
- Nematic and Hexatic Phases: are uniform and anisotropic
- Uniform fluids: break no spatial symmetries

HTS: Lattice effects $\Rightarrow$ breaking of (discrete) point group symmetries
If lattice effects are weak (high temperatures) $\Rightarrow$ continuous symmetries essentially recovered
2DEG in GaAs heterostructures $\Rightarrow$ continuous symmetries

Electronic Liquid Crystal Phases in HTS

Four Phases

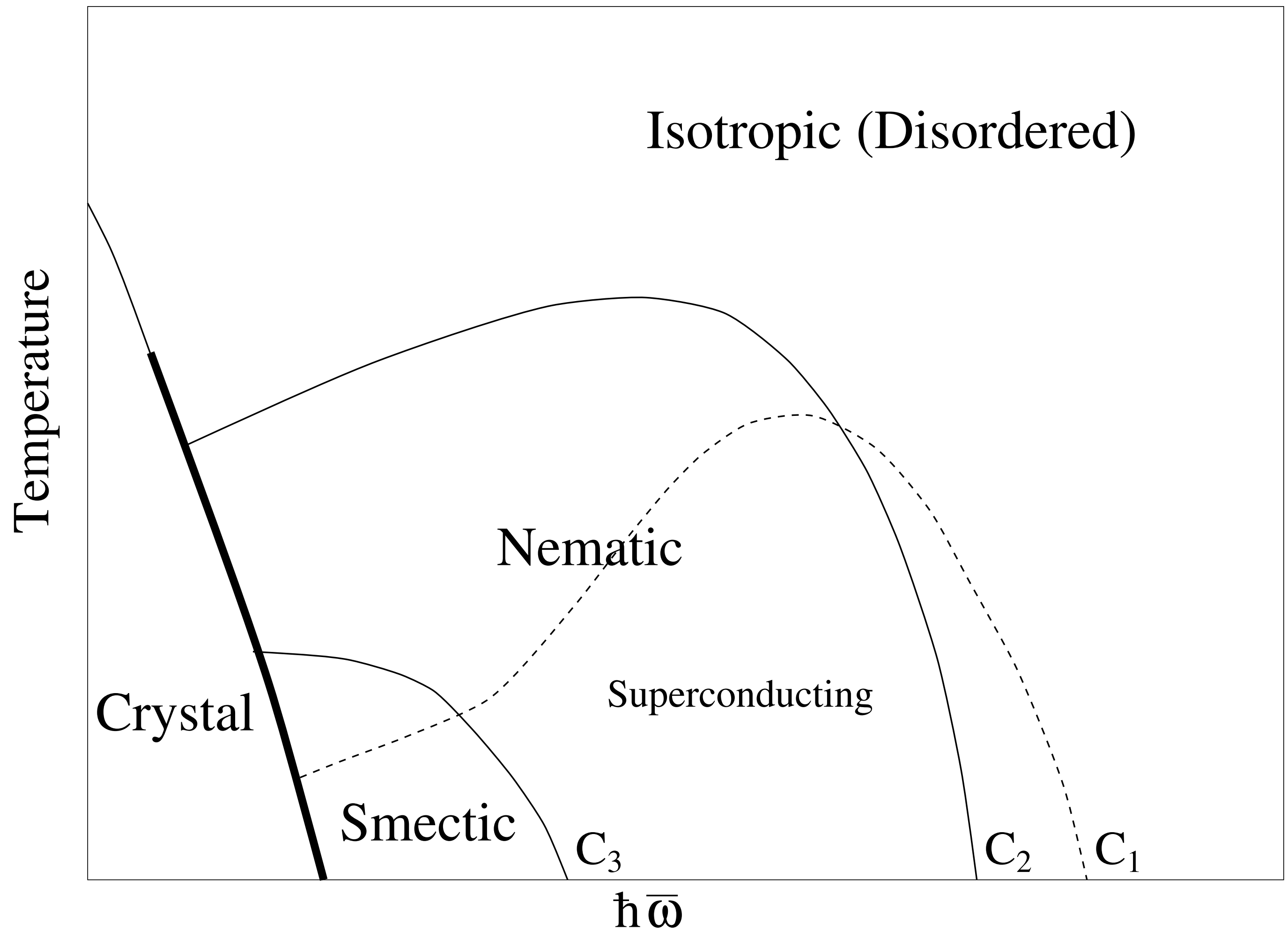
- Liquid: it is isotropic, breaks no spacial symmetries; it is either a conductor or a superconductor

- Nematic: lattice effects reduce the symmetry to a rotations by $\pi/2$ (" Ising"); unbroken translation and reflection symmetries; anisotropic liquid with a preferred axis

- Smectic: breaks translation symmetry only in one direction but liquid-like on the other; Stripe phase; (infinite) anisotropy of conductivity tensor

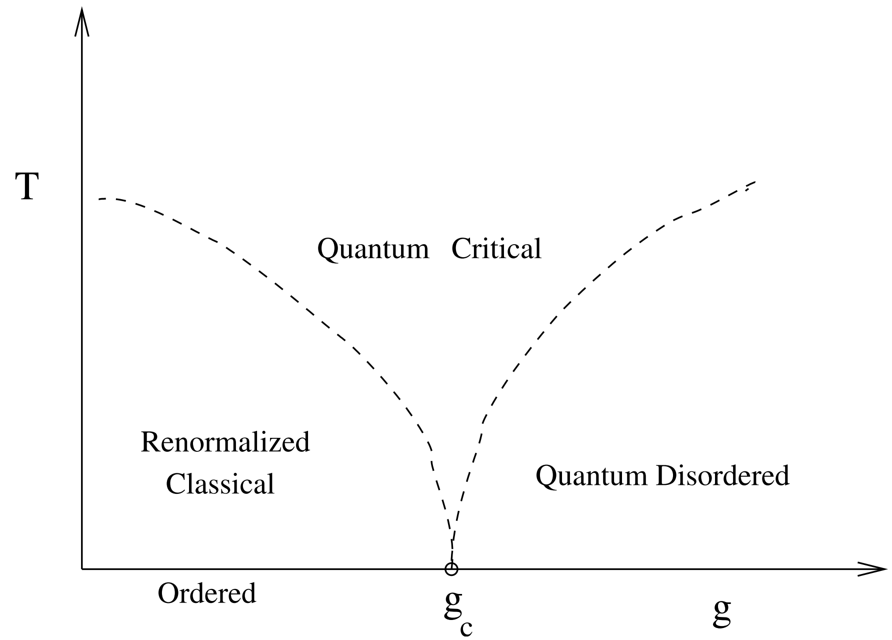
- Crystal(s): electron solids ("CDW"); insulating states.

Schematic Phase Diagram of Doped Mott Insulators



$\hbar\bar{\omega}$ measures transverse quantum fluctuations of the stripes. Systems with "large" coupling to lattice displacements (e.g. manganites) are "more classical" than systems with "primarily" electronic correlations (e.g. cuprates); nickelates lie in-between. Transverse stripe fluctuations enhance pair-tunneling and superconductivity.

Quantum Critical Point and Fluctuating Order



Electronic Liquid Crystal Phases may be detected by X-ray and neutron scattering (both are hard to do).

Local probes can be used to detect “local order”.

NMR, NQR, μSR and STM are quasi-static probes

They only work if the “fluctuating order” is pinned on the time scale of these experiments

Order Parameters for Charge Ordered States

Smectic (Stripe) State

- unidirectional CDW
- charge modulation $\Rightarrow$ charge stripe
- if it coexists with spin order $\Rightarrow$ spin stripe
- stripe state $\Rightarrow$ new Bragg peaks of the electron density at

$$\vec{k} = \pm \vec{Q}_{\text{ch}} = \pm \frac{2\pi}{\lambda_{\text{ch}}} \widehat{e}_x$$

- spin stripe $\Rightarrow$ magnetic Bragg peaks at

$$\vec{k} = \vec{Q}_s = (\pi, \pi) \pm \frac{1}{2} \vec{Q}_{\text{ch}}$$

- Order Parameter: $\langle n_{\vec{Q}_{\text{ch}}} \rangle$, Fourier component of the electron density at $\vec{Q}_{\text{ch}}$.

Nematic Order

Nematic State: broken rotational invariance but uniform

If the smectic (stripe) state melts (quantum mechanically or thermally) $\Rightarrow$ Local stripe ordered regions fluctuate

To detect broken rotational symmetry alone we need any quantity transforming like a traceless symmetric tensor

Useful definition in $D = 2$

$$\mathcal{Q}_{\vec{k}} = \frac{S(\vec{k}) - S(\mathcal{R}\vec{k})}{S(\vec{k}) + S(\mathcal{R}\vec{k})}$$

where

$$S(\vec{k}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S(\vec{k}, \omega)$$

$\mathcal{R} =$ rotation by $\pi/2$

Fluctuating Order near a Quantum Critical Point

Consider a system in its quantum disordered phase near a QCP *e. g.* charge order is absent as $T \to 0$ (other types of order may survive).

$S_{\text{ch}}(\vec{k}, \omega)$ for $\vec{k} \approx \vec{Q}_{\text{ch}}$ measures collective fluctuations most sensitive to the QCP.

Scaling $\Rightarrow \xi \sim \ell^{-\nu}$ and $\tau \sim \ell^{-\nu z}$ ($\ell = g - g_c$: distance to the QCP)

Quantum Disordered Phase: $$\text{E}_{\text{G}} \sim \frac{\hbar}{\tau} \sim \ell^{\nu \text{z}}.$$

- $\hbar\omega > E_G \Rightarrow S_{\text{ch}}(\vec{k}, \omega)$ has a pole corresponding to a sharply defined excitation whose quantum numbers are dictated by the nature of the nearby ordered state for $\ell < 0$
- For $\hbar\omega > 3E_G \Rightarrow S_{\text{ch}}(\vec{k}, \omega)$ has a multi-particle continuum
- For $\hbar\omega \gtrsim E_G$ we probe the quantum critical regime where there are no sharply defined quasi-particles since the anomalous dimension $\eta \neq 0$
- The continuum has a branch cut whose dispersion resembles that of the Goldstone modes of the ordered state

Classical vs. Quantum Critical Behavior

Near a *classical* critical point dynamics and thermodynamics are not necessarily connected

Classical Fluctuation-Dissipation Theorem:

Structure Factor $\longrightarrow S(\vec{k}) = T\chi(\vec{k}) \longleftarrow$ Susceptibility

Growing peak in $S(\vec{k})$ at $\vec{Q}_{\mathsf{ch}}$ with width $|\vec{k} - \vec{Q}_{\mathsf{ch}}| \sim \xi^{-1}$ and $S(\vec{Q}_{\mathsf{ch}}) \sim |T - T_c|^{-\gamma}$ reflects stripe order near T_c

Near a *quantum* critical point, dynamics is linked to thermodynamics
Quantum Fluctuation-Dissipation Theorem:

$$\begin{aligned}\chi(\vec{k},\Omega=0) &= \int \frac{d\omega}{2\pi}\,\frac{\chi''(\vec{k},\omega)}{\omega}\\ S(\vec{k}) &= \int \frac{d\omega}{2\pi}\,\mathrm{coth}\left(\frac{\hbar\omega}{2k_BT}\right)\,\chi''(\vec{k},\omega)\end{aligned}$$

- the largest contribution to $S(\vec{k})$ comes from the multi-particle continuum at large ω and it is small

- the largest contribution to $\chi(\vec{k},\Omega=0)$ comes from low ω

Near a QCP, $\chi''(\vec{k},\omega)$ scales: $\quad S(\vec{Q}_{\mathrm{ch}}) \sim \tau^{-1}\chi(\vec{Q}_{\mathrm{ch}},\Omega=0)$

$$\Rightarrow \begin{cases}\chi(\vec{Q}_{\mathrm{ch}},\Omega=0) \sim \ell^{-\gamma} & \text{strong singularity}\\ S(\vec{Q}_{\mathrm{ch}}) \sim \ell^{-\nu(2-z-\eta)} & \text{weak singularity}\end{cases}$$

Weak disorder makes life simpler!

Pure system: $S(\vec{k},\omega)$ has no information for $\hbar\omega \lesssim E_G$ and static experiments see nothing

Low quenched disorder, $V_{\text{disorder}} \sim E_G$ leads to important effects

- transfer of spectral weight to $\hbar\omega \lesssim E_G$, including $\omega \to 0$
- low frequency structure of $S(\vec{k},\omega)$ is largest for $\vec{k}$ where $S_{\text{pure}}(\vec{k},\omega)$ is large $\Rightarrow \vec{k} \sim \vec{Q}_{\text{ch}}$
- slow modes are most affected by weak disorder
- Lesson: weak disorder $\longrightarrow$ quasi-elastic peaks of $S_{\text{ch}}(\vec{k},\omega)$
- disorder eliminates the spectral gap and affects weakly $S_{\Omega}(\vec{k})$

Response functions for charge ordered states

How can we measure a response function for charge ordered states?

- Susceptibility: $V_{\vec{k}}$ small non-uniform potential
  $\Rightarrow \langle n_{\vec{k}} \rangle = \chi(\vec{k}) V_{\vec{k}}$
- for $|\vec{k} - \vec{Q}_{\text{ch}}| \lesssim \xi^{-1}$ and $E_G \lesssim V_{\vec{Q}_{\text{ch}}}$
  $\Rightarrow \langle n_{\vec{Q}_{\text{ch}}} \rangle \sim |V_{\vec{Q}_{\text{ch}}}|^{1/\delta}$, $\delta^{-1} = (d - 2 + \eta)/2$

STM: sensitive to the local DOS $\mathcal{N}(\vec{r}, E)$

- Pure system: $\mathcal{N}(\vec{r}, E) = \mathcal{N}_0(E)$
- Weak disorder: $\mathcal{N}(\vec{k}, E) = \chi_{\text{DOS}}(\vec{k}, E) V_{\vec{k}}$,
  $\mathcal{N}(\vec{k}, E) = \text{F. T.} \quad [\mathcal{N}(\vec{r}, E)]$

$$\chi_{\text{DOS}}(E, \vec{k}) = \int d\vec{r}\, dt\, d\tau\, e^{iEt - i\vec{k}\cdot\vec{r}} \theta(\tau) \langle [\{\Psi^\dagger_\sigma(\vec{r}, t+\tau), \Psi_\sigma(\vec{r}, \tau)\}, \hat{n}(\vec{0})] \rangle$$

$$\chi_{\text{ch}}(\vec{k}, \Omega = 0) = \int dE f(E)\, \chi_{\text{DOS}}(\vec{k}, E)$$

$f(E)$: Fermi function

Nematic order: $\chi(\vec{k}, \Omega = 0) = \chi(\mathcal{R}\vec{k}, \Omega = 0)$

$\longrightarrow$ we need a non-linear response

$$\mathcal{Q}_{\vec{k}} = \int d\vec{p}\, \chi_{\text{nem}}(\vec{k}; \vec{p}) [V_{\vec{p}} - V_{\mathcal{R}[\vec{p}]}][V_{-\vec{p}} + V_{-\mathcal{R}[\vec{p}]}] + \ldots$$

One-dimensional Luttinger Liquid at low T

- quantum critical CDW system with $z = 1$

- Luttinger parameters:

$$\begin{cases} K_c \leq 1, & \text{(repulsive interactions)} \\ K_s = 1 & \text{(spin rotation invariance)} \end{cases}$$

  no electron-like quasiparticles:
  spin-charge-separated solitons with $v_c \gtrsim v_s$

- Charge Susceptibility:
  $\chi_{\text{ch}}(2k_F + q) \sim |q|^{K_c - 1} \longrightarrow \infty$ as $q \to 0$

- Charge Density Structure Factor:

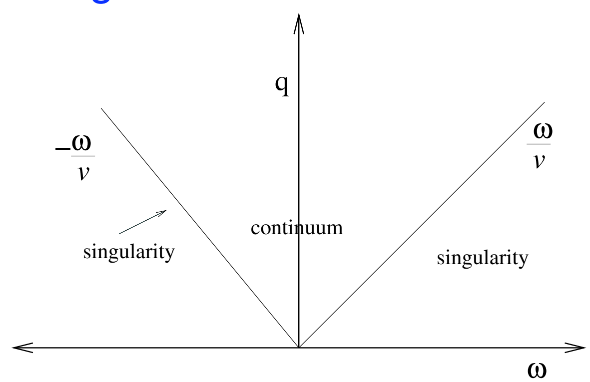
$$S(k, \omega) = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx \; \mathcal{S}(x, t) e^{ikx - i\omega t}$$

  $\mathcal{S}(r, t) = \mathcal{S}_0(r, t) + [e^{i2k_{\text{F}}r} \mathcal{S}_{2k_{\text{F}}}(r, t) + \text{c.c.}] + [e^{i4k_{\text{F}}r} \mathcal{S}_{4k_{\text{F}}}(r, t) + \text{c.c.}]$
  quantum criticality $\Rightarrow$ scaling $\Rightarrow$

$$S(2k_F + q, \omega) = \frac{1}{v_c} \left(\frac{D}{\hbar v_c q} \right)^a \Phi_{2k_F} \left(\frac{\omega}{v_c q}, \frac{\hbar\omega}{k_B T} \right)$$

  $a = 1 - K_c$ and $\Phi(x, y)$ depends on K_c and v_c / v_s

Spectrum of Charged Excitations in a 1D Luttinger Liquid



- As $q \to 0$

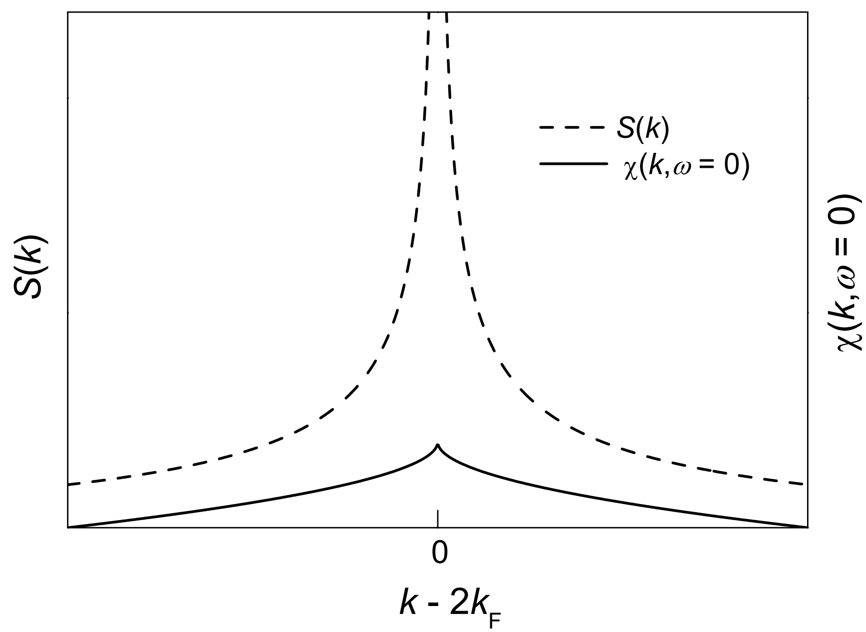
$$S(2k_{\mathsf{F}}+q,\omega) \sim \frac{1}{v_c}\left(\frac{D}{\hbar v_{\mathsf{C}} q}\right)^a \theta\left(\frac{\omega^2}{v^2q^2}-1\right)\left[\frac{\omega^2}{v^2q^2}-1\right]^{\frac{K_{\mathsf{C}}-1}{2}}$$

- For ω fixed, as $q \to \omega/v$, singularity where an ordered CDW has phason (Goldstone) modes

$$S(2k_{\mathsf{F}}+q,\omega) \sim [\,\omega - v|q|\,]^{(K_c-1)/2}$$

Charge Structure Factor $S(k)$ and Static Susceptibility $\chi(k)$ for a Luttinger Liquid

$$v_s = v_c,\ K_c = 0.5$$



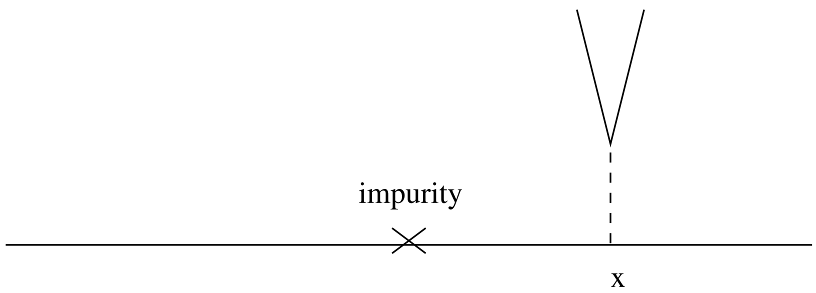
Equal-time structure factor is singular but finite

$$S(2k_F + q) \sim A - A' \left(q\alpha/2\right)^{K_c}$$

while

$$\chi(2k_F + q) \sim |q|^{-(1-K_c)}$$

STM of a 1D Luttinger Liquid and Charge Order



Consider the effects of a *single* impurity in a TLL
$K_c < 1$ (repulsive) $\Rightarrow V_{2k_F} \equiv \Gamma$ is relevant (Kane and Fisher)
$\Rightarrow \exists$ crossover scale $T_K \sim \Gamma^{2/(1-K_c)}$

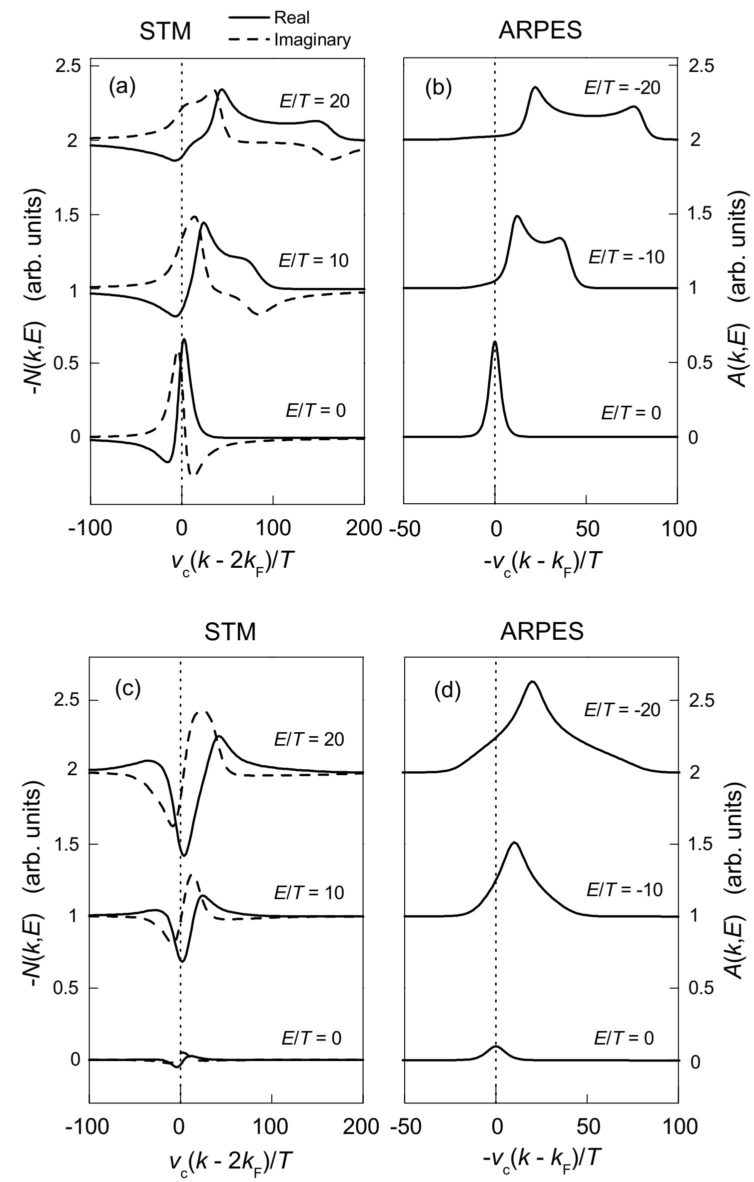
$$\text{RG flow} \ : \begin{cases} E \gg T_K & \to \Gamma \to 0 \\ E \ll T_K & \to \Gamma \to \infty \end{cases}$$

At low energies the impurity $\Leftrightarrow$ boundary condition *current*$= 0$
Behavior of the $2k_F$ component of the LDOS for $E \ll T_K$:

$$N(q + 2k_{\mathsf{F}}, E) = \frac{\hbar B}{E}\left(\frac{E}{D}\right)^{2b} \Phi\left(\frac{2E}{\hbar v_{\mathsf{C}} q}, \frac{E}{k_B T}\right)$$

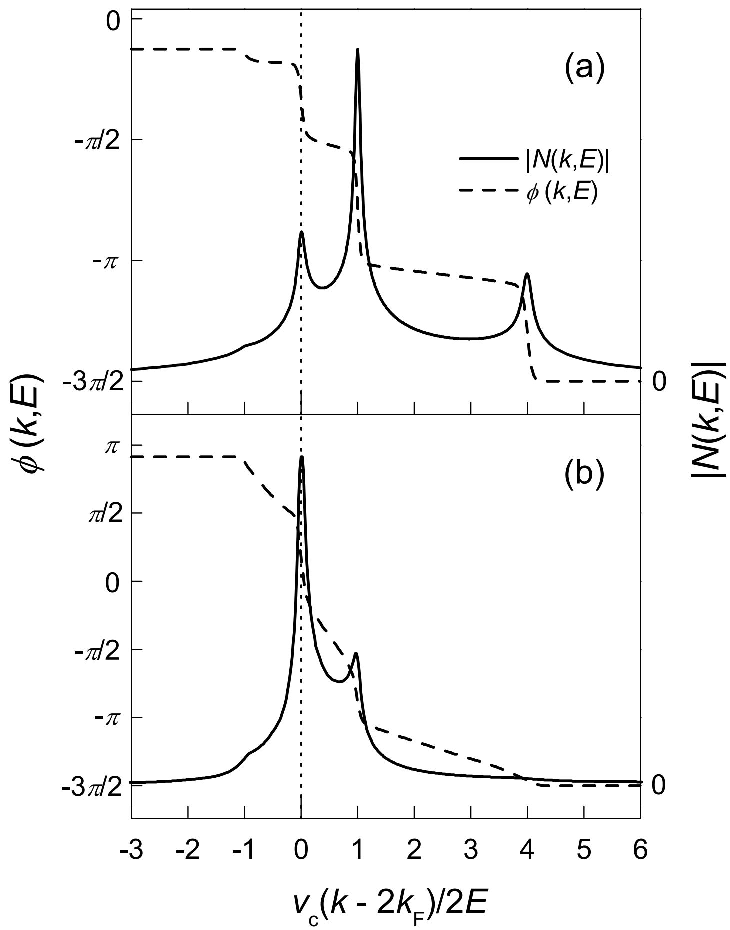
$$b = (1 - K_c)^2/4K_c$$

Thermally Scaled STM (left) and ARPES (right) spectra



$v_C/v_S = 4$ and $K_S = 1$; $K_C = 0.5$ (a,b), $K_C = 0.17$ (c,d).

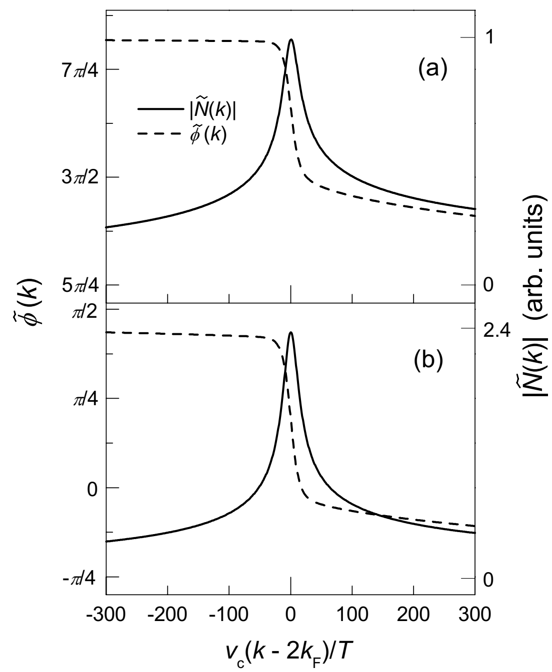
Low Temperature STM Spectra near $2k_F$



$E/k_BT = 100$; a) $K_c = 0.5$, b) $K_c = 0.17$.

$$N(k,E) = |N(k,E)|\, e^{i\phi(k,E)}$$

Thermally Scaled Energy-integrated DOS $\tilde{N}(k, T_K)$ of a 1D Luttinger Liquid with an impurity



$v_c/v_s = 4$ and $K_s = 1$
a) $K_c = 0.5$ and b) $K_c = 0.17$.

$$\tilde{N}(2k_F + q, T_K) \sim \left(\frac{T_K}{v_c}\right)\left(\frac{T_K}{D}\right)^{-2b}\left(\frac{T_K}{v_c q}\right)^{-(1-K_c)/2}$$

Weakly interacting 2DEG: Fermi Liquid

DOS susceptibility (in 2D): $\epsilon_{\vec{k}} = \hbar^2 \vec{k}^2 / 2m$

$$\chi^0_{\mathrm{DOS}}(E, \vec{k}) = \frac{m}{\pi \hbar^2} \frac{\theta(\epsilon_{\vec{k}} - 4E)}{\sqrt{\epsilon_{\vec{k}} \left(\epsilon_{\vec{k}} - 4E\right)}}$$

It is singular (in 2D) at $\epsilon_{\vec{k}} \to 4E$ (closed curves, not peaks!)
$\Rightarrow$ DOS modulations induced by weak disorder are quite different from the effects of a proximate CDW QCP

2D Charge Susceptibility $\to$ weak singularity as $|\vec{q}| \to 2k_F$

$$\chi_0(\vec{q}) = \frac{m}{2\pi \hbar^2} \left(1 - \theta(q - 2k_{\mathrm{F}}) \sqrt{1 - \frac{4k_{\mathrm{F}}^2}{q^2}} \right)$$

$\to$ Friedel Oscillations

Fermi Liquid (RPA)

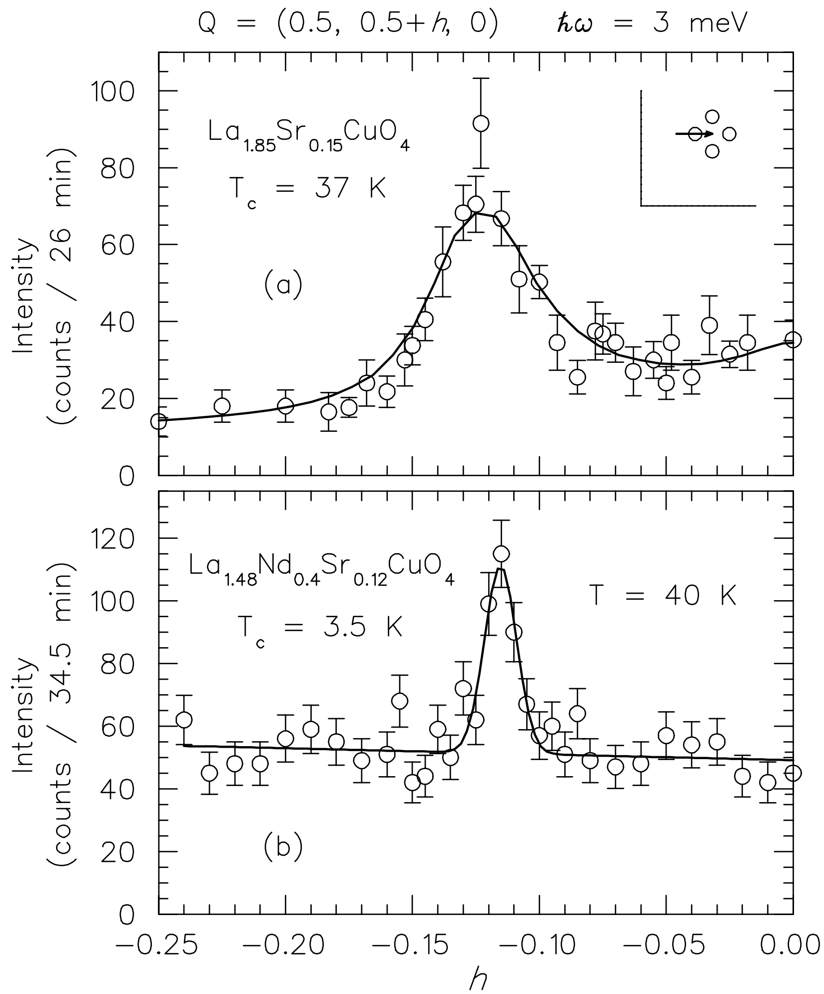
$$\begin{aligned} \chi_{\mathrm{ch}}(\vec{k}) &= [1 - U_{\vec{k}} \chi_0(\vec{k})]^{-1} \chi_0(\vec{k}) \\ \chi_{\mathrm{DOS}}(\vec{k}, E) &= [1 - U_{\vec{k}} \chi_0(\vec{k})]^{-1} \chi^0_{\mathrm{DOS}}(\vec{k}, E). \end{aligned}$$

Near a CDW QCP

$$\chi_{\mathrm{DOS}}(\vec{k}, E) \approx [1 - U_{\vec{k}} \chi_0(\vec{k})]^{-1} \chi^0_{\mathrm{DOS}}(\vec{Q}_{\mathrm{ch}}, E).$$

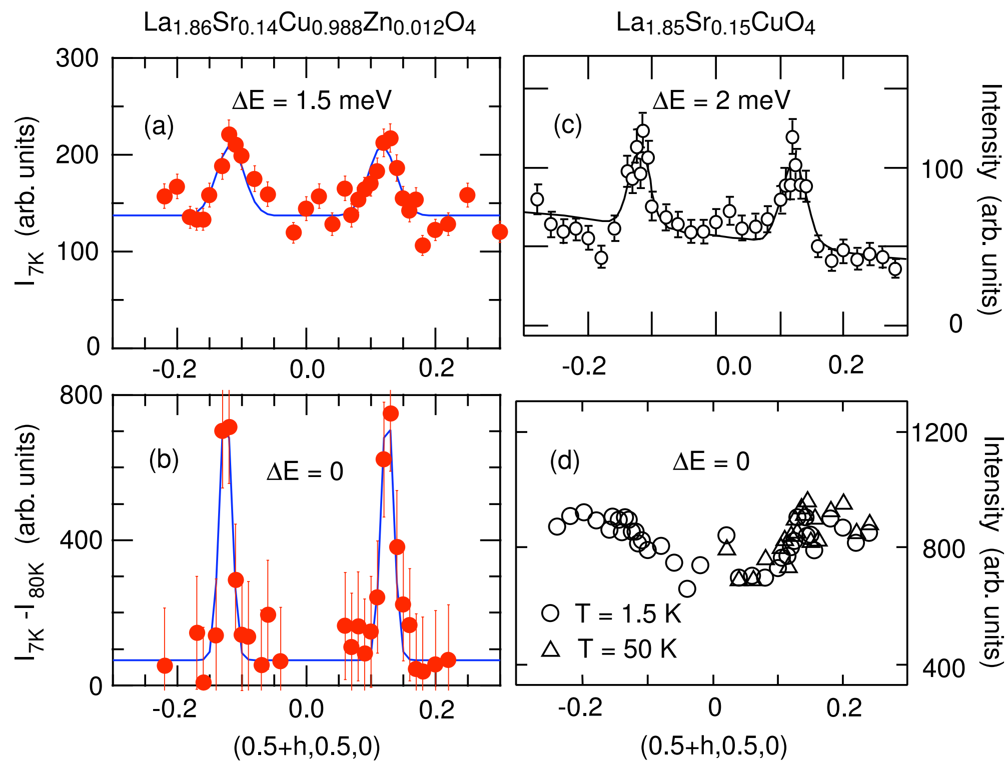
Prefactor: singular at $\vec{Q}_{\mathrm{ch}}$

Diffraction from Stripes with and without LRO



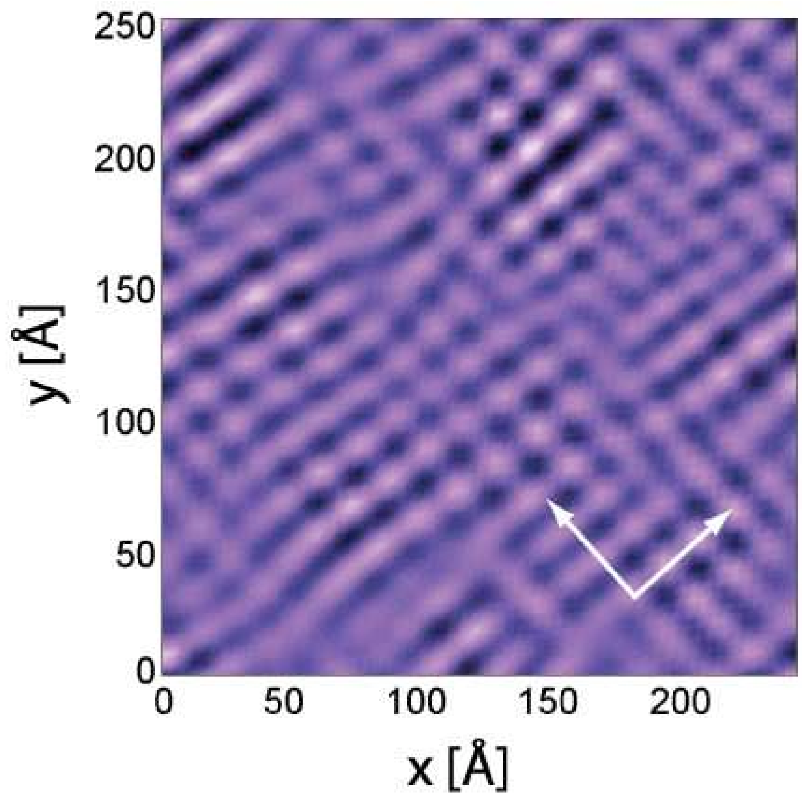
Comparison of constant-energy scans at $\hbar\omega = 3$ meV through an incommensurate magnetic peak (along path shown in inset) for (a) $La_{1.85}Sr_{0.15}CuO_4$ and (b) $La_{1.48}Nd_{0.4}Sr_{0.12}CuO_4$. Both scans are at $T = 40$ K $> T_c$; J. Tranquada, N. Ichikawa and S. Uchida, Phys. Rev. B **59**, 14712 (1999).

Induced stripe order in LSCO by Zn impurities

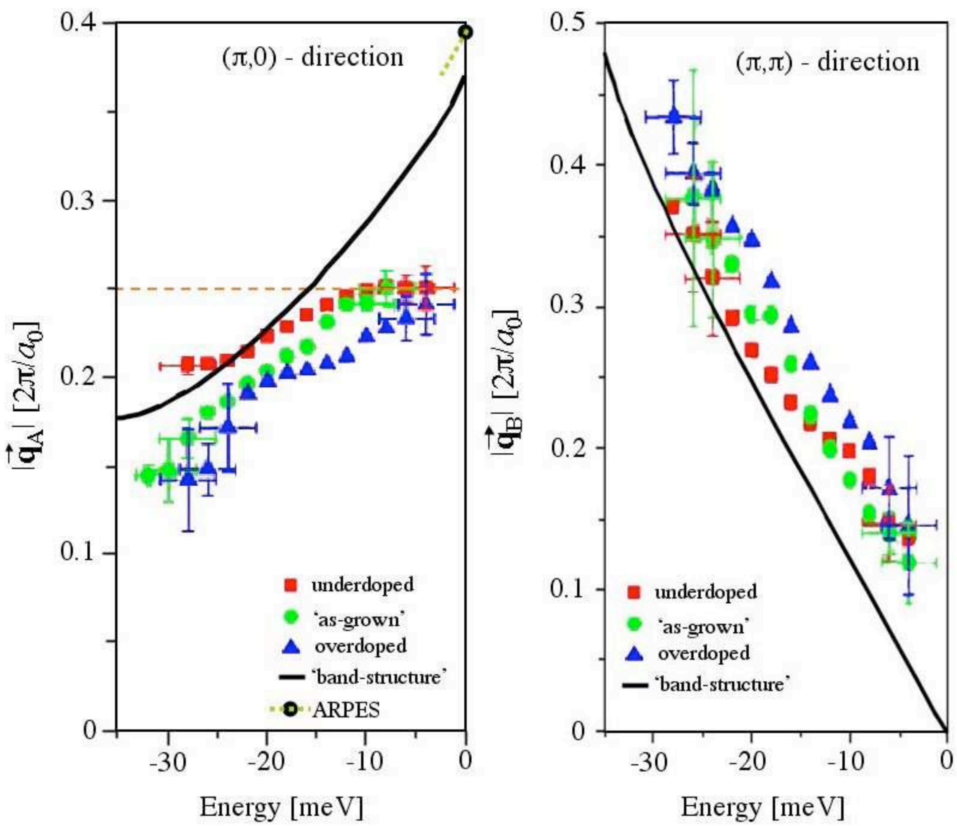


Comparison of magnetic scattering measurements with and without Zn; K. Himura *et. al.*, Phys. Rev. B **59**, 6517 (1999); K. Hirota, Physica C **357-360**, 61 (2001); K. Yamada *et. al.*, Phys. Rev. B **57**, 6165 (1998). all scans are along $Q = (\frac{1}{2} + h, \frac{1}{2}, 0)$, measured in reciprocal lattice units. (a) Scan at $E = 1.5$ meV and $T = 7$ K, and (b) difference between elastic scans measure at 7 K and 80 K, both for $La_{1.86}Sr_{0.14}Cu_{0.988}Zn_{0.012}O_4$ ($T_c = 19$ K). (c) Scan at $E = 2$ meV and $T = 38$ K, and (d) elastic scans at $T = 1.5$ K (circles) and 50 K (triangles), for $La_{1.85}Sr_{0.15}CuO_4$ ($T_c = 38$ K).

STM evidence for local stripe/nematic charge order in superconducting $Bi_2Sr_2CaCu_2O_{8+\delta}$



STM quasiparticle spectrum in superconducting $Bi_2Sr_2CaCu_2O_{8+\delta}$

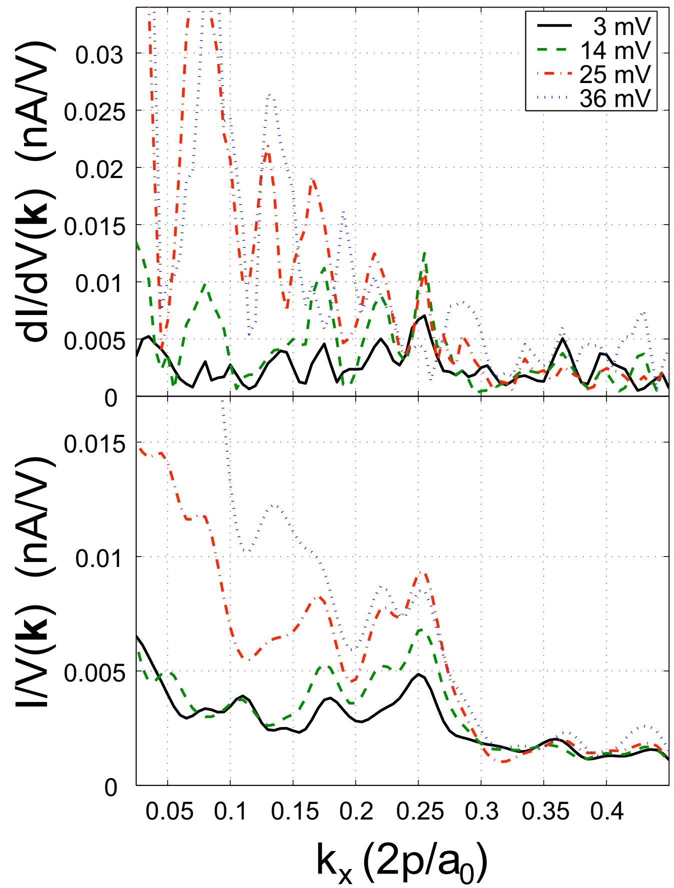


Data from J. E. Hoffman *et. al.*, Science **297**, 1148-1151 (2002)

black curves: fits with Norman's ARPES band structure with a d-wave gap

Both quasiparticles and induced order are seen!

Integrating the spectrum up to some energy reveals static induced local order



Real part of the FT of the LDOS in BSCCO along the CuO bond direction for $T \ll T_c$ (data from Kapitulnik's group). The same effect was found **above** T_c by Yazdani's group.

Similar striking effects were found earlier by J. Hoffman *et. al.*, Science **295**, 466 (2002), who studied order induced by a vortex

Conclusions

- Lesson No. 1: It is the low ω part of $S(\vec{k},\omega)$ which is most sensitive to charge ordering. Best way to investigate this question: integrate $S(\vec{k},\omega)$ over a window $\sim E_G/\hbar$.

- Lesson No.2: Weak disorder pins fluctuations and induces local order without disturbing intrinsic correlations $\longrightarrow$ tool to observe the nearby ordered state.

- Lesson No.3: Dispersing features of STM spectra give information on elementary excitations but do not imply the existence of well defined quasiparticles.

- Lesson No.4: Non-dispersing features of the STM spectra yields information of the nature of the nearby charge ordered state (stripe and/or nematic)

- Lesson No.5: In 2D interference effects produce peaks along curves in $\vec{k}$ space in manner dictated by the quasiparticles. Pinning of incipient order produces peaks at well defined points in $\vec{k}$ space leading to non-dispersing features in the STM spectra.