Liquid Crystal Phases in Strongly Correlated Systems

Eduardo Fradkin

Department of Physics

University of Illinois

Collaborators

Steven Kivelson (UCLA)

Vadim Oganesyan (Princeton)

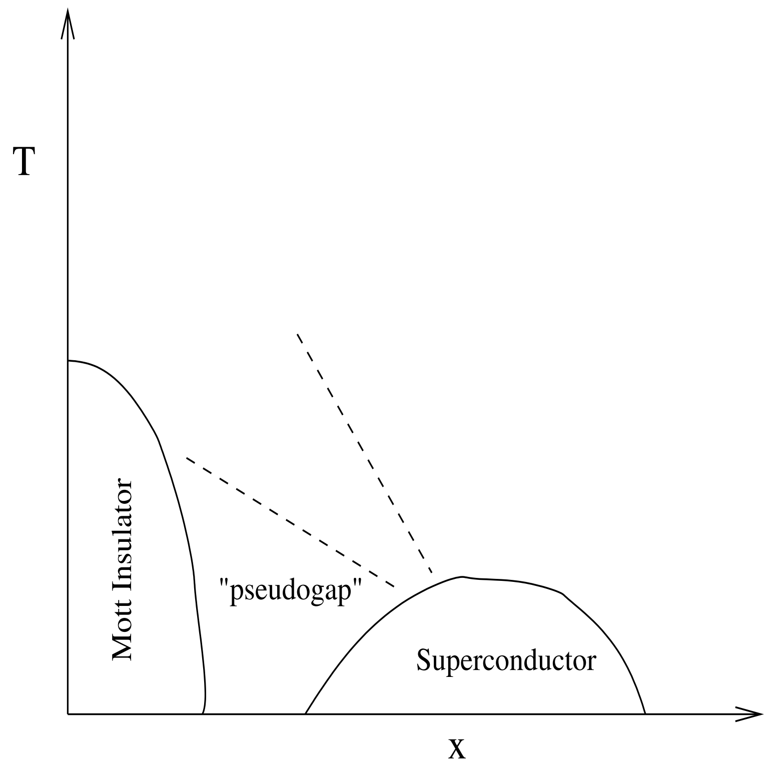
In memory of Victor J. Emery

KITP Conference, May 19, 2003

Outline

1. Liquid Crystal Phases in Strongly Correlated Systems
2. Anisotropic Phase of the 2DEG in large magnetic fields
3. Phases of quantum smectics
4. Quantum nematic order at weak and strong coupling
5. "Fluctuating order" and quantum disorder
6. Disorder as a tool to observe (fluctuating) order
7. Examples of fluctuating order in
   - 1DEG at $T = 0$
   - Weakly interacting 2DEG
8. Detection of fluctuating smectic (or stripe) and nematic order in High T_c superconductors: Neutron scattering, STM, and other probes.
9. Conclusions

Phase Diagram of the High T_c Superconductors



- Evidence for stripe charge order in underdoped HTS (LSCO and YBCO) (Tranquada, Ando)

- Evidence of coexistence of stripe charge order and superconductivity in LSCO and YBCO (Mook, Tranquada)

- STM Experiments suggest existence of short range stripe order and possibly broken rotational symmetry in BSCO (Kapitulnik, Davis, Yazdani)

Charge and Spin Order in Doped Mott Insulators

Many strongly correlated (quantum) systems exhibit spatially inhomogeneous and anisotropic phases.

Stripe phases of cuprate superconductors

Stripe phases of manganites and nickelates

Anisotropic transport in 2DEG in large magnetic fields

Common underlying physical mechanism:

$$\text{Competition} \longrightarrow \left\{ \begin{array}{l} \text{effective short range attractive forces} \\ \text{long(er) range repulsive (Coulomb) interactions} \end{array} \right.$$

uniform gapped state not allowed $\Rightarrow$ spatial inhomogeneity

Examples in classical systems: blockcopolymers, ferrofluids, etc.

Astrophysical examples: “Pasta Phases” of neutron stars

Analogues in lipid bilayers intercalated with DNA

Soft Quantum Matter

or

Quantum Soft Matter

Electron Liquid Crystal Phases

S. Kivelson, E. Fradkin, V. Emery, Nature **393**, 550 (1998)

Doping a Mott insulator: inhomogeneous phases due to the competition between phase separation and strong correlations

- Crystal Phases: break all continuous translation symmetries and rotations
- Smectic (Stripe) phases: break one translation symmetry and rotations
- Nematic and Hexatic Phases: are uniform and anisotropic
- Uniform fluids: break no spatial symmetries

HTS: Lattice effects $\Rightarrow$ breaking of point group symmetries

If lattice effects are weak (high temperatures) $\Rightarrow$ continuous symmetries essentially recovered

2DEG in GaAs heterostructures $\Rightarrow$ continuous symmetries

Electronic Liquid Crystal Phases in HTS

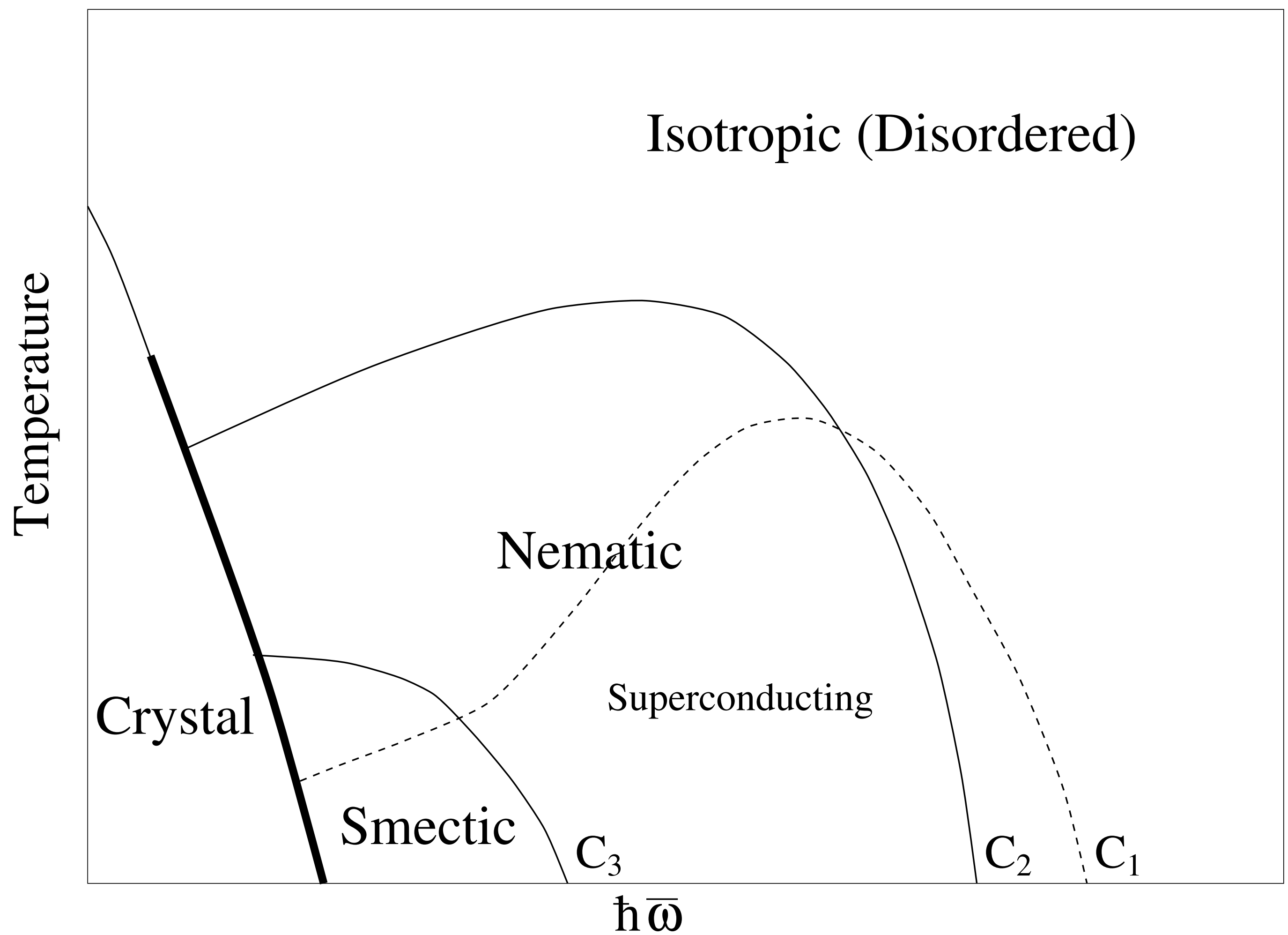
- Liquid: Isotropic, breaks no spacial symmetries; either a conductor or a superconductor

- Nematic: Lattice effects reduce the symmetry to a rotations by $\pi/2$ (“ Ising”); translation and reflection symmetries are unbroken; it is an anisotropic liquid with a preferred axis

- Smectic: breaks translation symmetry only in one direction but liquid-like on the other; Stripe phase; (infinite) anisotropy of conductivity tensor

- Crystal(s): electron solids (“CDW”); insulating states.

Schematic Phase Diagram of Doped Mott Insulators



$\hbar\bar{\omega}$ measures transverse zero-point stripe fluctuations of the stripes.

Systems with “large” coupling to lattice displacements (e. g. manganites) are “more classical” than systems with “primarily” electronic correlations (e. g. cuprates); nickelates lie in-between.

Transverse stripe fluctuations enhance pair-tunneling and superconductivity.

Order Parameters for Charge Ordered States

Smectic (Stripe) State

- unidirectional CDW
- charge modulation $\Rightarrow$ charge stripe
- if it coexists with spin order $\Rightarrow$ spin stripe
- stripe state $\Rightarrow$ new Bragg peaks of the electron density at

$$\vec{k} = \pm \vec{Q}_{\text{ch}} = \pm \frac{2\pi}{\lambda_{\text{ch}}} \hat{e}_x$$

- spin stripe $\Rightarrow$ magnetic Bragg peaks at

$$\vec{k} = \vec{Q}_s = (\pi, \pi) \pm \frac{1}{2} \vec{Q}_{\text{ch}}$$

- Order Parameter: $\langle n_{\vec{Q}_{\text{ch}}} \rangle$, Fourier component of the electron density at $\vec{Q}_{\text{ch}}$.

Nematic Order

Nematic State: broken rotational invariance but uniform

If the smectic (stripe) state melts (either quantum mechanically or by thermal fluctuations) $\Rightarrow$ Local stripe ordered regions fluctuate

To detect broken rotational symmetry alone we need any

quantity transforming like a traceless symmetric tensor

For example, in $D = 2$ one can use

$$S(\vec{k}) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} S(\vec{k}, \omega)$$

to construct

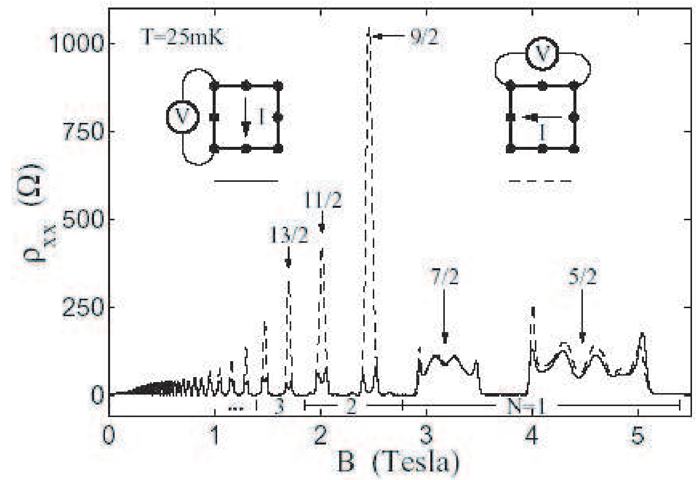
$$\mathcal{Q}_{\vec{k}} = \frac{S(\vec{k}) - S(\mathcal{R}\vec{k})}{S(\vec{k}) + S(\mathcal{R}\vec{k})}$$

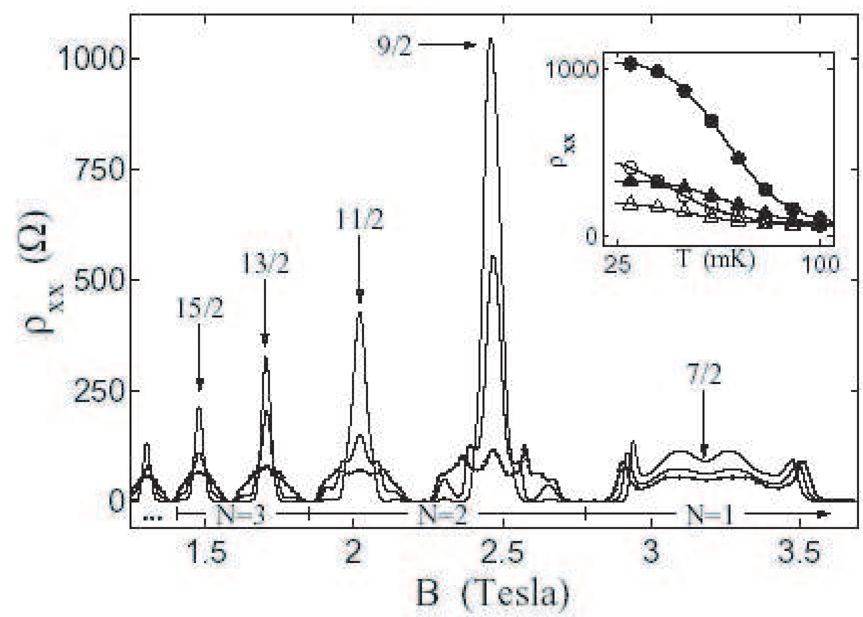
$S(\vec{k}, \omega)$; dynamic structure factor
$\mathcal{R}$ = rotation by $\pi/2$.

Transport: use the resistivity tensor to construct $\mathcal{Q}$

$$\mathcal{Q} = \frac{\rho_{\mathrm{xx}} - \rho_{\mathrm{yy}}}{\rho_{\mathrm{xx}} + \rho_{\mathrm{yy}}}$$

Transport Anisotropy in 2DEG in large B Fields



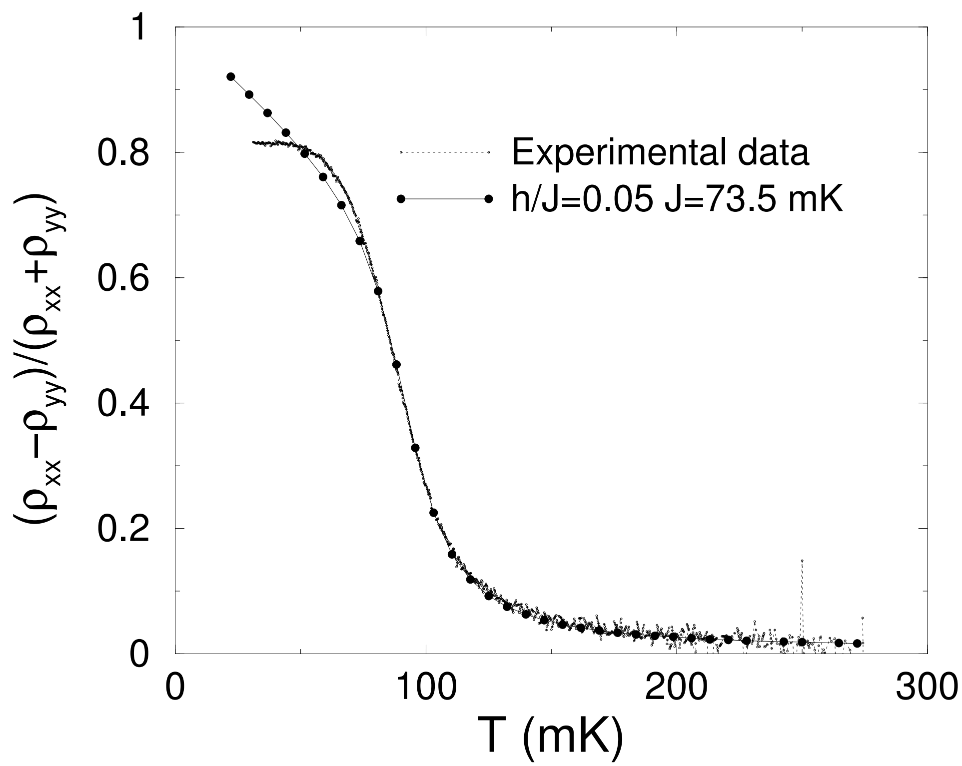


M. Lilly *et.al.*, Phys. Rev. Lett. **82**, 394 (1999)

Summary of the experiments

- High mobility samples (33×10^7) and long mean-free path (0.5 mm!)
- $I - V$ curves are linear at low V and non-linear at high V; no thresholds in the anisotropic regime, sharp thresholds in the IQHE reentrant regime.
- No noise in the anisotropic regime; broad band noise observed in IQHE reentrant regime.
- It behaves like an anisotropic fluid with a temperature dependent anisotropy!

The 2DEG behaves like a Nematic fluid!



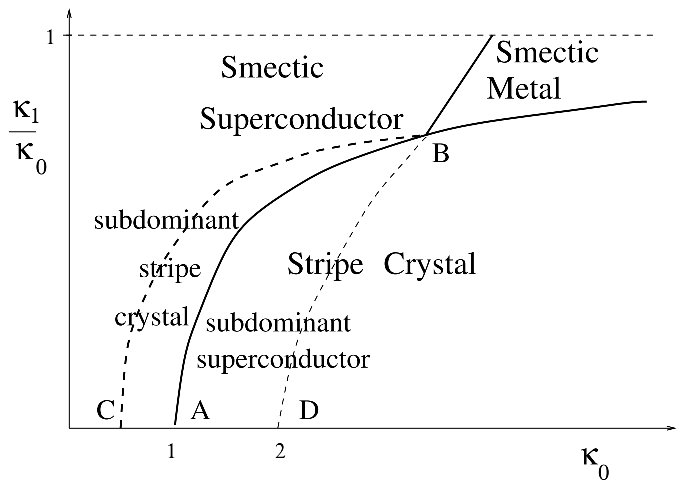
Monte Carlo simulation of a classical 2D XY model with coupling J and external field h, on a 100×100 lattice, fitted to the data of Lilly and coworkers, at $\nu = 9/2$ (after deconvoluting geometric effects).
Best fit: $J = 73mK$, $h = 0.05J = 3.5mK$ and $T_c = 65mK$.

E. Fradkin, S. A. Kivelson, E. Manousakis and K. Nho, PRL. **84**, 1982 (2000).
K. B. Cooper, M. P. Lilly, J. P. Eisenstein, L. N. Pfeiffer and K. W. , West, PRB**65**, 241313/1-4 (2002)

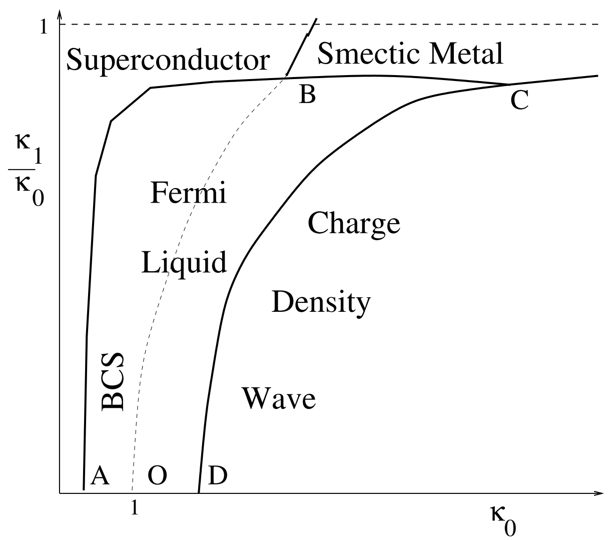
Phases of Smectic (Stripe) States

- Stripe states are quasi-one dimensional phases
- Charge degrees of freedom are “confined” to one dimensional structures, a.k.a “stripes”
- Stripe states can be described as arrays of “sliding” Luttinger liquids
- Forward scattering interactions are marginal $\longrightarrow$ many Luttinger parameters
- Physics depends on the possible existence of a spin gap
- Pair tunneling $\longrightarrow$ superconductivity
- Inter-stripe $2k_F$ couplings $\longrightarrow$ crystals (CDW)
- Electron tunneling (if unsuppressed by a spin gap) $\longrightarrow$ 2D nematic Fermi liquids

Phase diagram with a spin gap



Phase diagram without a spin gap



V. J. Emery, E. Fradkin, S. A. Kivelson and T. C. Lubensky, PRL **85**, 2160 (2000)
A. Vishwanath and D. Carpentier, PRL **86**, 676 (2001)

A Nematic "Fermi Liquid"

V. Oganesyan, S. A. Kivelson, and E. Fradkin, PRB **64**, 195109 (2001)

- We can think of a quantum nematic either as a melted smectic (strong coupling) or as a Fermi surface instability (weak coupling)

- The nematic order parameter for two-dimensional Fermi fluid is the 2×2 symmetric traceless tensor

$$\hat{\mathbf{Q}}(x) \equiv -\frac{1}{k_F^2}\Psi^\dagger(\vec{r}) \begin{pmatrix} \partial_x^2 - \partial_y^2 & 2\partial_x\partial_y \\ 2\partial_x\partial_y & \partial_y^2 - \partial_x^2 \end{pmatrix} \Psi(\vec{r}),$$

- In the nematic phase

$$\mathbf{Q} \equiv < \hat{\mathbf{Q}} > \equiv Q e^{2i\theta} = Q_{11} + iQ_{12} \neq 0$$

- The Fermi surface spontaneously distorts into an ellipse (Pomeranchuk instability) with an eccentricity $\propto Q$

- this state is uniform and breaks rotational invariance (mod π)

- It is trivial to construct hexatic states as well

Simple Fermi liquid model

$$H = \int d\vec{r}\, \Psi^\dagger(\vec{r})\epsilon(\vec{\nabla})\Psi(\vec{r}) + \frac{1}{4}\int d\vec{r}\int d\vec{r}\,' F_2(\vec{r}-\vec{r}\,')\mathrm{Tr}[\hat{\mathbf{Q}}(\vec{r})\hat{\mathbf{Q}}(\vec{r}\,')]$$

$$\epsilon(\vec{k}) = v_F q[1 + a(\frac{q}{k_F})^2], \qquad \text{where} \quad q \equiv |\vec{k}| - k_F$$

$$F_2(\vec{r}) = \int \frac{d^2k}{(2\pi)^2} e^{i\vec{q}\cdot\vec{r}} \frac{F_2}{1+\kappa F_2 q^2}$$

F_2 is a Landau parameter.

Landau theory of the Landau theory

Landau energy density functional:

$$\mathcal{V}[\mathbf{Q}] = E(\mathbf{Q}) - \frac{\tilde{\kappa}}{4}\mathrm{Tr}[\mathbf{Q}\mathbf{D}\mathbf{Q}] - \frac{\tilde{\kappa}'}{4}\mathrm{Tr}[\mathbf{Q}^2\mathbf{D}\mathbf{Q}] + \ldots$$

$$E(\mathbf{Q}) = E(\mathbf{0}) + \frac{A}{4}\mathrm{Tr}[\mathbf{Q}^2] + \frac{B}{8}\mathrm{Tr}[\mathbf{Q}^4] + \ldots$$

$A = \frac{1}{2N_F} + F_2$ N_F is the density of states at the Fermi surface, $E_F \equiv v_F k_F$ is the Fermi energy, and $B = \frac{3aN_F|F_2|^3}{8E_F^2}$.

If $A < 0 \Rightarrow$ nematic phase

Phase Diagram

- an isotropic Fermi liquid phase
- a nematic non-Fermi liquid phase

separated by a quantum critical point at $2N_F F_2 = -1$

Physics of the Nematic Phase:

- Transverse Goldstone boson which is generically overdamped except for $\phi = 0, \pm\pi/4, \pm\pi/2$ (symmetry directions) where it is underdamped
- Anisotropic (Drude) Transport

$$\frac{\rho_{xx}-\rho_{yy}}{\rho_{xx}+\rho_{yy}} = \frac{1}{2}\frac{m_y - m_x}{m_y + m_x} = \frac{\text{Re } Q}{E_F} + \mathcal{O}(Q^3)$$

- Quasiparticle scattering rate (one loop): In general

$$\Sigma''(\epsilon, \vec{k}) = \frac{\pi}{\sqrt{3}} \frac{(\kappa k_F^2)^{1/3}}{\kappa N_F} \left| \frac{k_x k_y}{k_F^2} \right|^{4/3} \left| \frac{\epsilon}{2 v_F k_F} \right|^{2/3} + \ldots$$

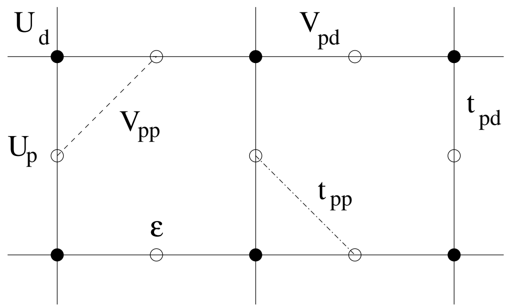
  Along a symmetry direction:

$$\Sigma''(\epsilon) = \frac{\pi}{3 N_F \kappa} \frac{1}{(\kappa k_F^2)^{1/4}} \left| \frac{\epsilon}{v_F k_F} \right|^{3/2} + \ldots$$

- The Nematic phase is a Non-Fermi liquid with “nodal excitations”!
- At the quantum phase transition its behavior is the same as fermions coupled to a fluctuating gauge field (c.f. P. A. Lee and collaborators)
- Preliminary work indicates that it favors an instability to extended s-wave superconductivity (H-Y Kee and Y-B Kim, 2002)

Nematic States in the Strongly Coupled Emery Model of a CuO plane

S.A. Kivelson, E.Fradkin and T. Geballe , cond-mat/0302163



Energetics of the 2D $Cu - O$ model in the strong coupling limit:

$$t_{pd}/U_p, t_{pd}/U_d, t_{pd}/V_{pd}, t_{pd}/V_{pp} \to 0$$

$$U_d > U_p \gg V_{pd} > V_{pp} \qquad \text{and} \qquad t_{pp}/t_{pd} \to 0$$

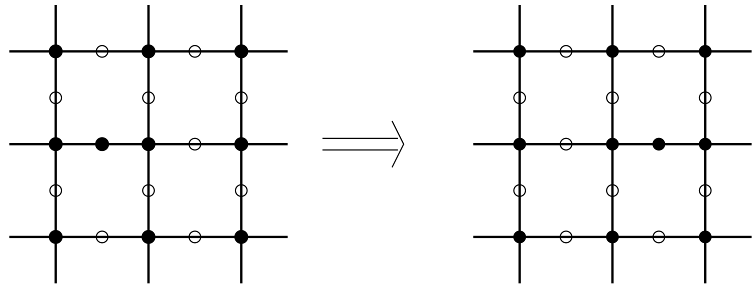
as a function of hole doping $x > 0$ ($x = 0 \Leftrightarrow$ half-filling)

Energy to add one hole: $\mu \equiv 2V_{pd} + \epsilon$

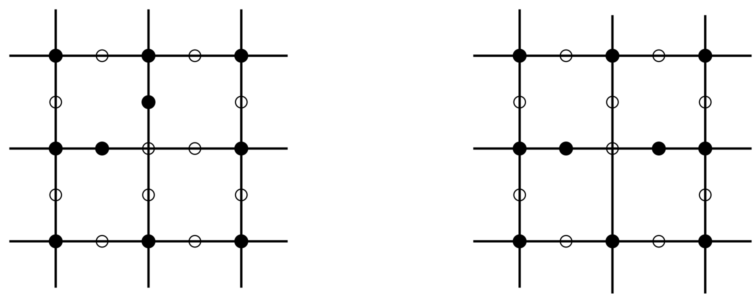
Energy of two holes on nearby O sites: $\mu + V_{pp} + \epsilon$

Effective One-Dimensional Dynamics at Strong Coupling

In the strong coupling limit, and at $t_{pp} = 0$, the motion of an extra hole is strongly constrained. The following is an allowed move which takes two steps.



The final and initial states are degenerate, and their energy is $E_0 + \mu$



a) b)

- Intermediate state for the hole to turn a corner; it has energy $E_0 + \mu + V_{pp} \Rightarrow t_{\text{eff}} = \frac{t_{pd}^2}{V_{pp}} \ll t_{pd}$

- Intermediate state for the hole to continue on the same row; it has energy $E_0 + \mu + \epsilon \Rightarrow t_{\text{eff}} = \frac{t_{pd}^2}{\epsilon}$

Physics of the Nematic State

- The ground state at $x=0$ is an antiferromagnetic insulator

- Doped holes behave like one-dimensional spinless fermions

$$H_c = -t \sum_j [c_j^\dagger c_{j+1} + h.c.] + \sum_j [\epsilon_j \widehat{n}_j + V_{pd} \widehat{n}_j \widehat{n}_{j+1}]$$

- at $x = 1$ it is a Nematic insulator, and it is also insulating at other commensurate cases, *e. g.* $x = 1/2$

- the ground state for $x \to 0$ and $x \to 1$ is a uniform array of 1D Luttinger liquids $\Rightarrow$ it is an Ising Nematic Phase.

- This result follows from the observation that for $x \to 0$ the ground state energy of the nematic state is

$$E_{\text{nematic}} = E(x=0) + \Delta_c\, x + W\, x^3 + O(x^5)$$

where $\Delta_c = 2V_{pd} + \epsilon + \ldots$ and $W = \pi^2 \hbar^2 / 6m^*$, while the energy of the isotropic state is

$$E_{\text{isotropic}} = E(x=0) + \Delta_c\, x + (1/4) W\, x^3 + V_{\text{eff}}\, x^2$$

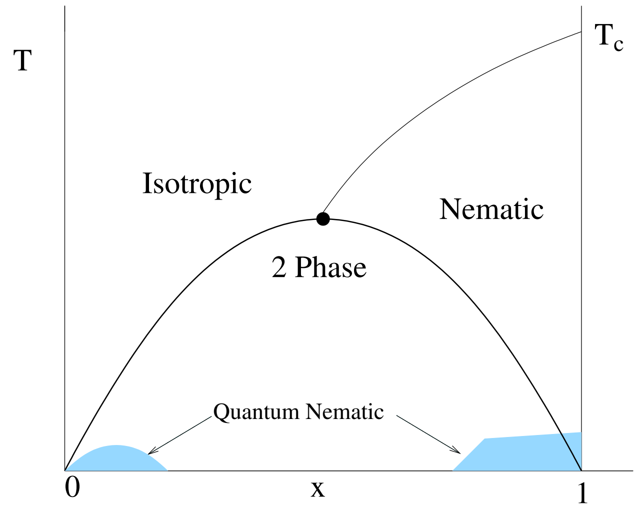
(V_{eff} is an effective coupling for holes on intersecting rows and columns)
$\Rightarrow E_{\text{nematic}} < E_{\text{isotropic}}$

- A similar argument holds for $x \to 1$.

- the density of mobile charge $\sim x$ but $k_F = (1-x)\pi/2$

- For $t_{pp} \neq 0$ this 1D state crosses over (most likely) to a 2D (Ising) Nematic Fermi liquid state.

Phase diagram in the strong coupling limit



- In the "classical" regime, $\epsilon/t_{pd} \to \infty$, with $U_d > \epsilon$, the doped holes are distributed on O sites at an energy cost μ per doped hole and an interaction $J = V_{pp}/4$ per neighboring holes on the O sub-lattice

- This is a classical lattice gas equivalent to a 2D classical Ising anti-ferromagnet with exchange J in a uniform "field" μ, and an effective magnetization (per O site) $m = 1 - x$

- The classical Ising antiferromagnet at temperature T and magnetization $m = 1 - x$ has the phase diagram of the figure.

- For $T \gg t_{pd}/\sqrt{t_{pd}^2 + \epsilon^2}$, the classical phase diagram holds even for $0 < t_{pd}/\epsilon < \infty$

- Quantum fluctuations lead to a similar phase diagram, except for the extra nematic phases at least near $x \sim 0$, $x \sim 1$ and $x \sim 1/2 \Rightarrow$ complex phase diagram in the quantum regime

“Fluctuating Order”

Ordered states are characterized by a spontaneously broke symmetry $\Rightarrow$ Order Parameter

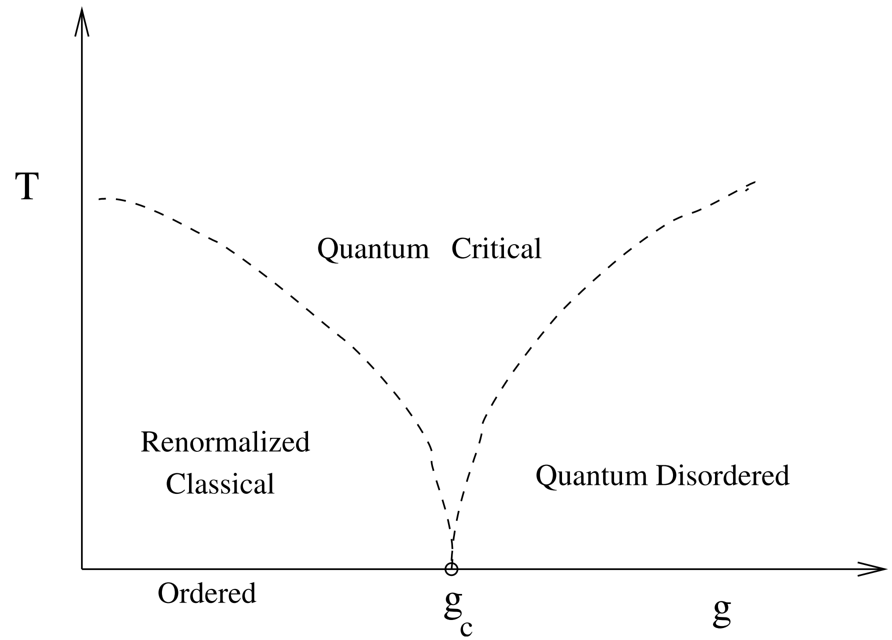
Order Parameter fluctuations grow as a (classical or quantum) critical point is approached $\Rightarrow$ Fluctuations are evidence for the proximate ordered state

quantum disordered state: fast fluctuations
$\tau \sim E_G^{-1}$ ($E_G \equiv$ Gap) $\Rightarrow$ unless $E_G \to 0$, τ is “short”

“Fluctuating Order” is an ill-defined concept **unless** the nearby ordered state is found

Detecting Ordered States: Best way
1) detect the broken symmetry
2) detect fluctuations, e. g. measure $S(\vec{k}, \omega)$

Quantum Critical Point and Fluctuating Order



Electronic Liquid Crystal Phases may be detected by X-ray and neutron scattering (both are hard to do).

Local probes can be used to detect “local order”.

NMR, NQR, μSR and STM are quasi-static probes

They only work if the “fluctuating order” is pinned on the time scale of these experiments

Fluctuating Order near a Quantum Critical Point

Consider a system in its quantum disordered phase near a QCP *e. g.* charge order is absent as $T \to 0$ (other types of order may survive).

$S_{\text{ch}}(\vec{k}, \omega)$ for $\vec{k} \approx \vec{Q}_{\text{ch}}$ measures collective fluctuations most sensitive to the QCP.

Scaling $\Rightarrow \xi \sim \ell^{-\nu}$ and $\tau \sim \ell^{-\nu z}$ ($\ell = g - g_c$: distance to the QCP)

Quantum Disordered Phase: $$\mathrm{E_G} \sim \frac{\hbar}{\tau} \sim \ell^{\nu \mathrm{z}}.$$

- $\hbar\omega > E_G \Rightarrow S_{\text{ch}}(\vec{k}, \omega)$ has a pole corresponding to a sharply defined excitation whose quantum numbers are dictated by the nature of the nearby ordered state for $\ell < 0$
- For $\hbar\omega > 3E_G \Rightarrow S_{\text{ch}}(\vec{k}, \omega)$ has a multi-particle continuum
- For $\hbar\omega \gtrsim E_G$ we probe the quantum critical regime where there are no sharply defined quasi-particles since the anomalous dimension $\eta \neq 0$
- The continuum has a branch cut whose dispersion resembles that of the Goldstone modes of the ordered state

Classical vs. Quantum Critical Behavior

Near a *classical* critical point dynamics and thermodynamics are not necessarily connected
Classical Fluctuation-Dissipation Theorem:

Structure Factor $\rightarrow S(\vec{k}) = T\chi(\vec{k}) \leftarrow$ Susceptibility

Growing peak in $S(\vec{k})$ at $\vec{Q}_{\text{ch}}$ with width $|\vec{k} - \vec{Q}_{\text{ch}}| \sim \xi^{-1}$ and $S(\vec{Q}_{\text{ch}}) \sim |T - T_c|^{-\gamma}$ reflects stripe order near T_c

Near a *quantum* critical point, dynamics is linked to thermodynamics

- largest contribution to $S(\vec{k})$ comes from the multi-particle continuum at large ω and it is small
- the largest contribution to $\chi(\vec{k})$ comes from low ω

Near a QCP, $\chi''(\vec{k}, \omega)$ scales: $S(\vec{Q}_{\text{ch}}) \sim \tau^{-1}\chi(\vec{Q}_{\text{ch}})$

$$\Rightarrow \begin{cases} \chi(\vec{Q}_{\text{ch}}) \sim \ell^{-\gamma} & \text{strong singularity} \\ S(\vec{Q}_{\text{ch}}) \sim \ell^{-\nu(2-z-\eta)} & \text{weak singularity} \end{cases}$$

Weak disorder makes life simpler!

Pure system: $S(\vec{k},\omega)$ has no information for $\hbar\omega \lesssim E_G$ and static experiments see nothing

Low quenched disorder, $V_{\text{disorder}} \sim E_G$ leads to important effects

- transfer of spectral weight to $\hbar\omega \lesssim E_G$, including $\omega \to 0$
- low frequency structure of $S(\vec{k},\omega)$ is largest for $\vec{k}$ where $S_{\text{pure}}(\vec{k},\omega)$ is large $\Rightarrow \vec{k} \sim \vec{Q}_{\text{ch}}$
- slow modes are most affected by weak disorder
- Lesson: weak disorder $\longrightarrow$ quasi-elastic peaks of $S_{\text{ch}}(\vec{k},\omega)$
- disorder eliminates the spectral gap and affects weakly $S_{\Omega}(\vec{k})$

Response functions for charge ordered states

How can we measure a response function for charge ordered states?

- Susceptibility: $V_{\vec{k}}$ small non-uniform potential
  $\Rightarrow \langle n_{\vec{k}} \rangle = \chi(\vec{k}) V_{\vec{k}}$
- for $|\vec{k} - \vec{Q}_{\text{ch}}| \lesssim \xi^{-1}$ and $E_G \lesssim V_{\vec{Q}_{\text{ch}}}$
  $\Rightarrow \langle n_{\vec{Q}_{\text{ch}}} \rangle \sim |V_{\vec{Q}_{\text{ch}}}|^{1/\delta}$, $\delta^{-1} = (d - 2 + \eta)/2$

STM: sensitive to the local DOS $\mathcal{N}(\vec{r}, E)$

- Pure system: $\mathcal{N}(\vec{r}, E) = \mathcal{N}_0(E)$
- Weak disorder: $\mathcal{N}(\vec{k}, E) = \chi_{\text{DOS}}(\vec{k}, E) V_{\vec{k}}$,
  $\mathcal{N}(\vec{k}, E) = \text{F. T. } [\mathcal{N}(\vec{r}, E)]$

$$\chi_{\text{DOS}}(E, \vec{k}) = \int d\vec{r}\, dt\, d\tau\, e^{iEt - i\vec{k}\cdot\vec{r}} \theta(\tau) \langle [\{\Psi^\dagger_\sigma(\vec{r}, t+\tau), \Psi_\sigma(\vec{r}, \tau)\}, \hat{n}(\vec{0})] \rangle$$

$$\chi_{\text{ch}}(\vec{k}, \Omega = 0) = \int dE f(E)\, \chi_{\text{DOS}}(\vec{k}, E)$$

$f(E)$: Fermi function
Nematic order: $\chi(\vec{k}, \Omega = 0) = \chi(\mathcal{R}\vec{k}, \Omega = 0)$
$\rightarrow$ we need a non-linear response

$$\mathcal{Q}_{\vec{k}} = \int d\vec{p}\, \chi_{\text{nem}}(\vec{k}; \vec{p}) [V_{\vec{p}} - V_{\mathcal{R}[\vec{p}]}][V_{-\vec{p}} + V_{-\mathcal{R}[\vec{p}]}] + \ldots$$

One-dimensional Luttinger Liquid at low T

- quantum critical CDW system with $z = 1$

- Luttinger parameters:

$$\begin{cases} K_c \leq 1, & \text{(repulsive interactions)} \\ K_s = 1 & \text{(spin rotation invariance)} \end{cases}$$

  no electron-like quasiparticles:
  spin-charge-separated solitons with $v_c \gtrsim v_s$

- Charge Susceptibility:
  $\chi_{\text{ch}}(2k_F + q) \sim |q|^{K_c - 1} \longrightarrow \infty$ as $q \to 0$

- Charge Density Structure Factor:

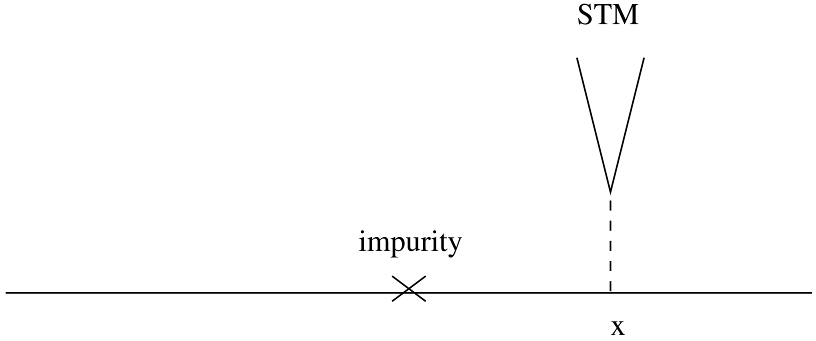
$$S(k,\omega) = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx \, \mathcal{S}(x,t) e^{ikx - i\omega t}$$

  $\mathcal{S}(r,t) = \mathcal{S}_0(r,t) + [e^{i2k_{\text{F}}r}\mathcal{S}_{2k_{\text{F}}}(r,t) + \text{c.c.}] + [e^{i4k_{\text{F}}r}\mathcal{S}_{4k_{\text{F}}}(r,t) + \text{c.c.}]$
  quantum criticality $\Rightarrow$ scaling $\Rightarrow$

$$S(2k_F + q, \omega) = \frac{1}{v_c}\left(\frac{D}{\hbar v_{\text{c}} q}\right)^a \Phi_{2k_F}\left(\frac{\omega}{v_c q}, \frac{\hbar\omega}{k_B T}\right)$$

  $a = 1 - K_c$ and $\Phi(x,y)$ depends on K_c and v_c/v_s

STM of a 1D Luttinger Liquid and Charge Order



Consider the effects of a *single* impurity in a TLL

$K_c < 1$ (repulsive) $\Rightarrow V_{2k_F} \equiv \Gamma$ is relevant (Kane and Fisher)

$\Rightarrow \exists$ crossover scale $T_K \sim \Gamma^{2/(1-K_c)}$

$$\text{RG flow} \; : \begin{cases} E \gg T_K & \to \Gamma \to 0 \\ E \ll T_K & \to \Gamma \to \infty \end{cases}$$

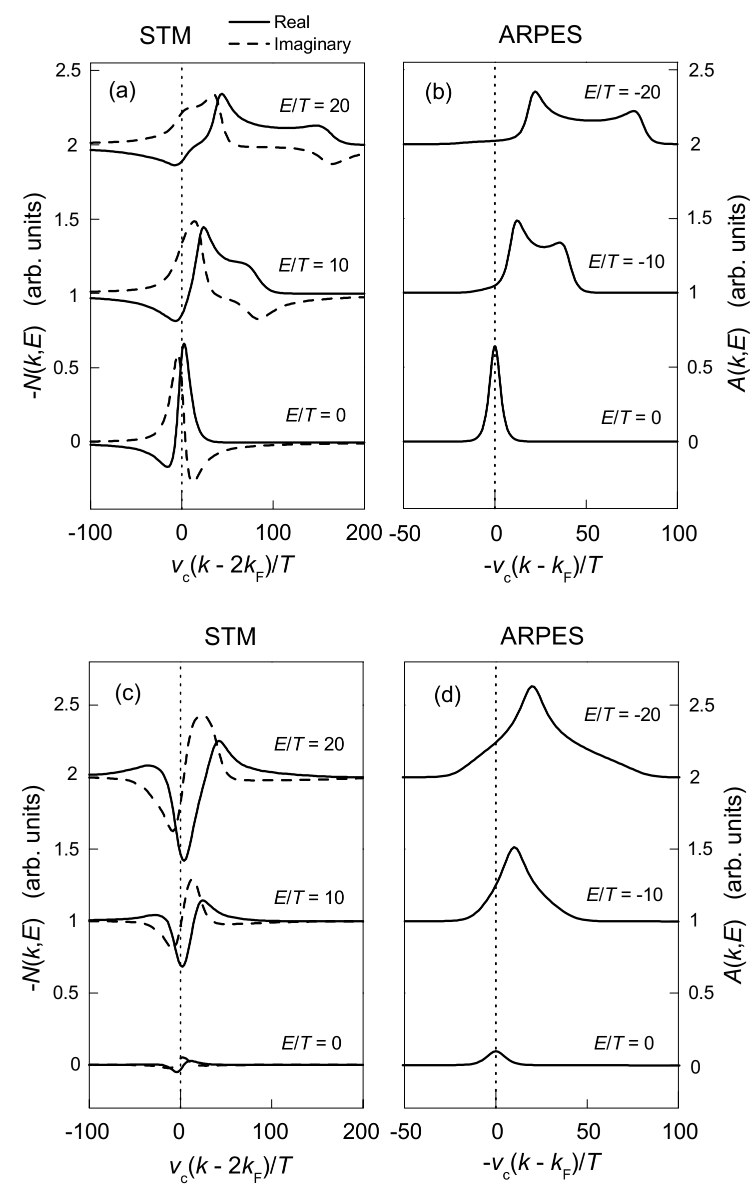
At low energies the impurity $\Leftrightarrow$ boundary condition *current*$= 0$

Behavior of the $2k_F$ component of the LDOS for $E \ll T_K$:

$$N(q + 2k_{\mathrm{F}}, E) = \frac{\hbar B}{E} \left(\frac{E}{D} \right)^{2b} \Phi \left(\frac{2E}{\hbar v_{\mathrm{c}} q}, \frac{E}{k_B T} \right)$$

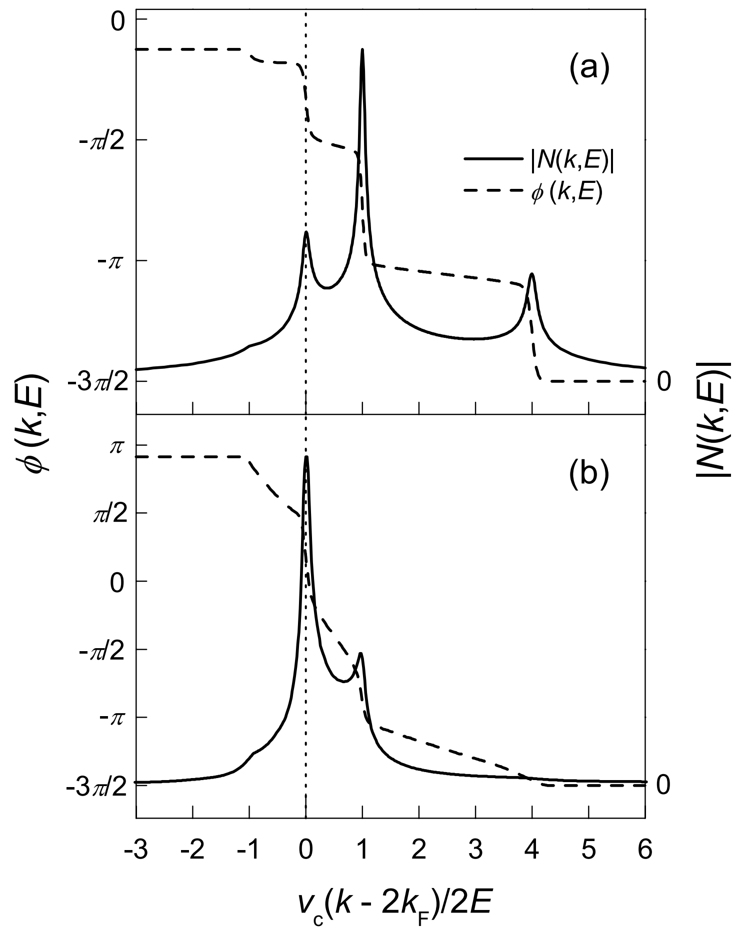
$$b = (1 - K_c)^2 / 4K_c$$

Thermally Scaled STM (left) and ARPES (right) spectra



$v_C/v_S = 4$ and $K_S = 1$; $K_C = 0.5$ (a,b), $K_C = 0.17$ (c,d).

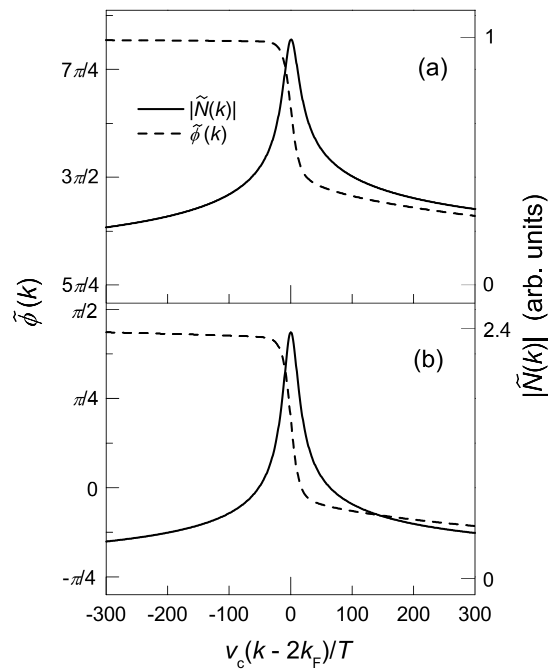
Low Temperature STM Spectra near $2k_F$



$E/k_BT = 100$; a) $K_c = 0.5$, b) $K_c = 0.17$.

$$N(k,E) = |N(k,E)|\, e^{i\phi(k,E)}$$

Thermally Scaled Energy-integrated DOS $\tilde{N}(k, T_K)$ of a 1D Luttinger Liquid with an impurity



$v_c/v_s = 4$ and $K_s = 1$
a) $K_c = 0.5$ and b) $K_c = 0.17$.

$$\tilde{N}(2k_F + q, T_K) \sim \left(\frac{T_K}{v_c}\right)\left(\frac{T_K}{D}\right)^{-2b}\left(\frac{T_K}{v_c q}\right)^{-(1-K_c)/2}$$

Weakly interacting 2DEG: Fermi Liquid

DOS susceptibility (in 2D): $\epsilon_{\vec{k}} = \hbar^2 \vec{k}^2 / 2m$

$$\chi^0_{\mathrm{DOS}}(E, \vec{k}) = \frac{m}{\pi\hbar^2} \frac{\theta(\epsilon_{\vec{k}} - 4E)}{\sqrt{\epsilon_{\vec{k}} \left(\epsilon_{\vec{k}} - 4E\right)}}$$

It is singular (in 2D) at $\epsilon_{\vec{k}} \to 4E$ (closed curves, not peaks!)
$\Rightarrow$ DOS modulations induced by weak disorder are quite different from the effects of a proximate CDW QCP

2D Charge Susceptibility $\to$ weak singularity as $|\vec{q}| \to 2k_F$

$$\chi_0(\vec{q}) = \frac{m}{2\pi\hbar^2} \left(1 - \theta(q - 2k_{\mathrm{F}}) \sqrt{1 - \frac{4k_{\mathrm{F}}^2}{q^2}} \right)$$

$\to$ Friedel Oscillations

Fermi Liquid (RPA)

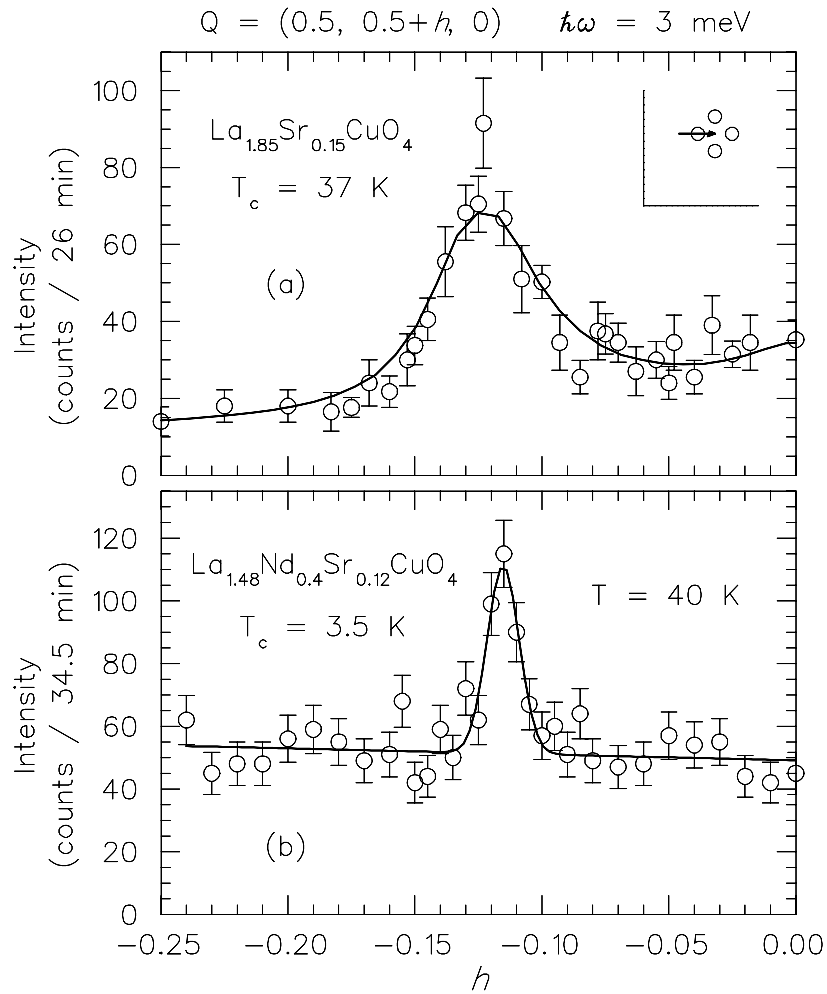
$$\begin{aligned} \chi_{\mathrm{ch}}(\vec{k}) &= \quad [1 - U_{\vec{k}} \chi_0(\vec{k})]^{-1} \chi_0(\vec{k}) \\ \chi_{\mathrm{DOS}}(\vec{k}, E) &= \quad [1 - U_{\vec{k}} \chi_0(\vec{k})]^{-1} \chi^0_{\mathrm{DOS}}(\vec{k}, E). \end{aligned}$$

Near a CDW QCP

$$\chi_{\mathrm{DOS}}(\vec{k}, E) \approx [1 - U_{\vec{k}} \chi_0(\vec{k})]^{-1} \chi^0_{\mathrm{DOS}}(\vec{Q}_{\mathrm{ch}}, E).$$

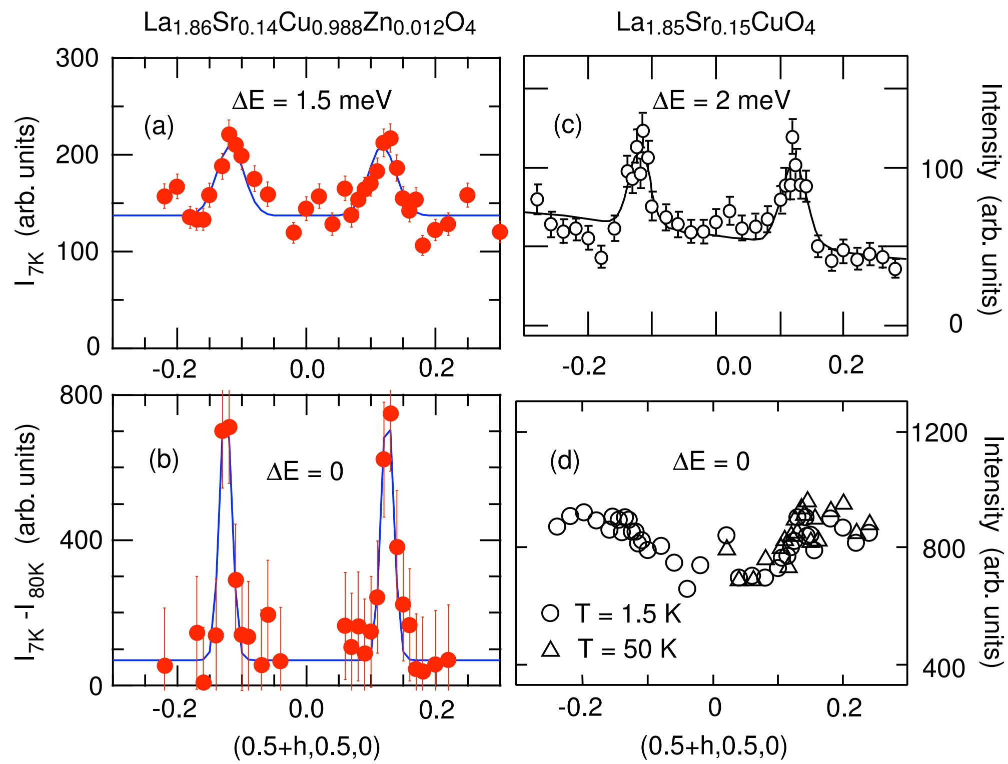
Prefactor: singular at $\vec{Q}_{\mathrm{ch}}$

Diffraction from Stripes with and without LRO



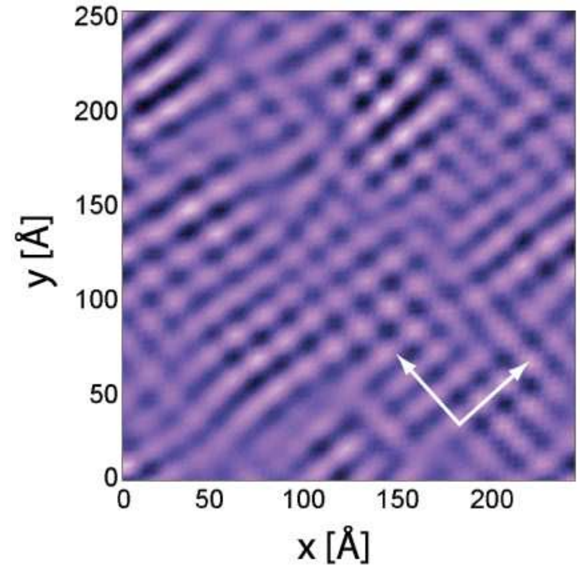
Comparison of constant-energy scans at $\hbar\omega = 3$ meV through an incommensurate magnetic peak (along path shown in inset) for (a) $La_{1.85}Sr_{0.15}CuO_4$ and (b) $La_{1.48}Nd_{0.4}Sr_{0.12}CuO_4$. Both scans are at $T = 40$ K $> T_c$; J. Tranquada, N. Ichikawa and S. Uchida, Phys. Rev. B **59**, 14712 (1999).

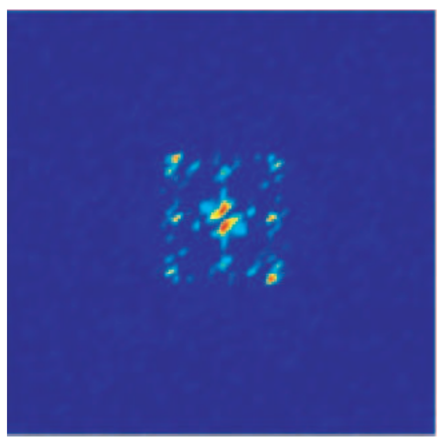
Induced stripe order in LSCO by Zn impurities



Comparison of magnetic scattering measurements with and without Zn; K. Himura *et. al.*, Phys. Rev. B **59**, 6517 (1999); K. Hirota, Physica C **357-360**, 61 (2001); K. Yamada *et. al.*, Phys. Rev. B **57**, 6165 (1998). all scans are along $Q = (\frac{1}{2} + h, \frac{1}{2}, 0)$, measured in reciprocal lattice units. (a) Scan at $E = 1.5$ meV and $T = 7$ K, and (b) difference between elastic scans measure at 7 K and 80 K, both for $La_{1.86}Sr_{0.14}Cu_{0.988}Zn_{0.012}O_4$ ($T_c = 19$ K). (c) Scan at $E = 2$ meV and $T = 38$ K, and (d) elastic scans at $T = 1.5$ K (circles) and 50 K (triangles), for $La_{1.85}Sr_{0.15}CuO_4$ ($T_c = 38$ K).

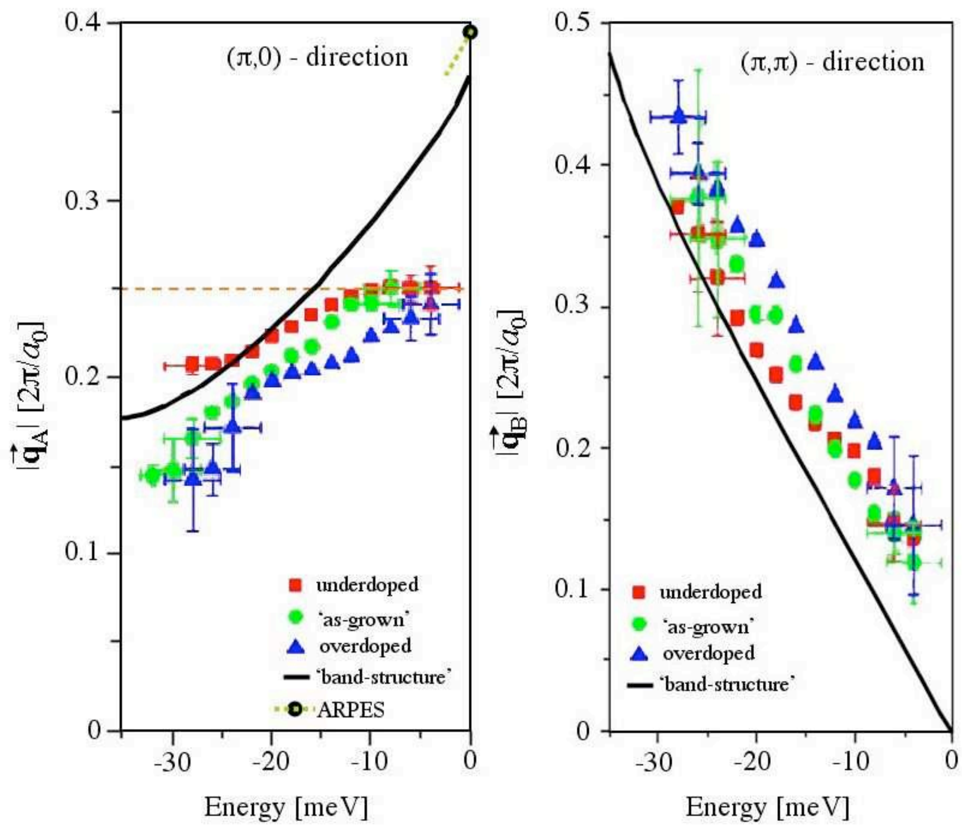
STM evidence for local stripe/nematic charge order in superconducting $Bi_2Sr_2CaCu_2O_{8+\delta}$





C. Howald *et. al.*, cond-mat/0201546

STM quasiparticle spectrum in superconducting $Bi_2Sr_2CaCu_2O_{8+\delta}$

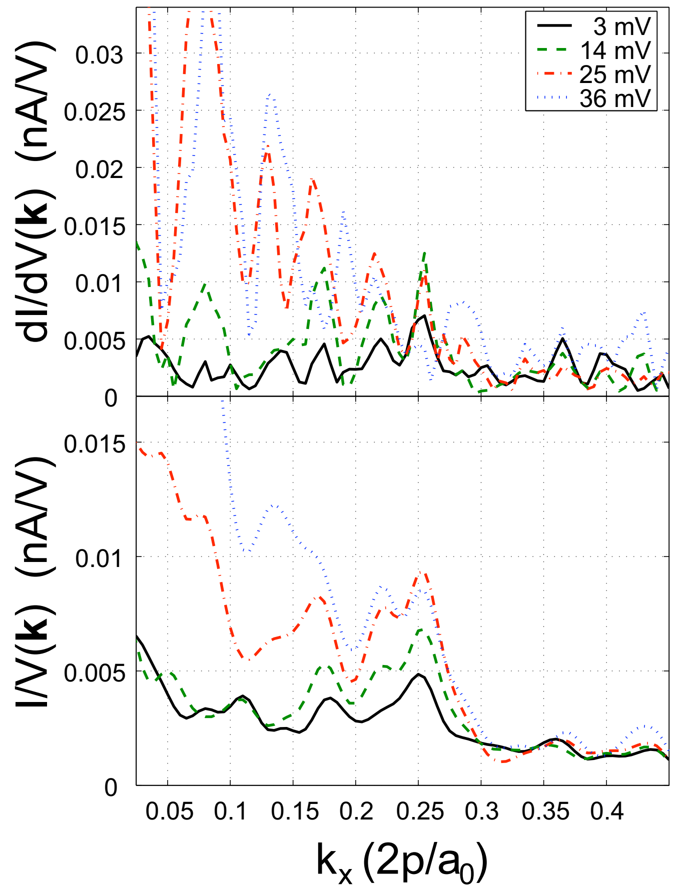


Data from J. E. Hoffman *et. al.*, Science **297**, 1148-1151 (2002)

black curves: fits with Norman's ARPES band structure with a d-wave gap

Both quasiparticles and induced order are seen!

Integrating the spectrum up to some energy reveals static induced local order



Real part of the FT of the LDOS in BSCCO along the CuO bond direction for $T \ll T_c$ (data from Kapitulnik's group). The same effect was found **above** T_c by Yazdani's group.

Similar striking effects were found earlier by J. Hoffman *et. al.*, Science **295**, 466 (2002), who studied order induced by a vortex

Conclusions

- Liquid Crystal phases are a generic feature of doped Mott insulators.
- We gave a phenomenological characterization of these phases
- Smectic (stripe) phases lead to phase diagrams with competing orders
- Nematic phases are non-Fermi liquid like (if unpinned by the lattice) and appear naturally in the strong coupling regime of simple strongly correlated models.
- We discussed the origin of the nematic phases from two points of view
  1. As an instability of a Fermi Liquid State
  2. As the strong coupling limit of the Emery Model
- We reviewed the concept of fluctuating (charge) order and discussed ways of measuring it
- We gave examples in the 1DEG and 2DEG of fluctuating charge order
- We discussed evidence for fluctuating charge order in neutron scattering and STM experiments in HTS