Electronic Liquid Crystal Phases in Strongly Correlated Systems

Lectures at the Les Houches Summer School, May 2009

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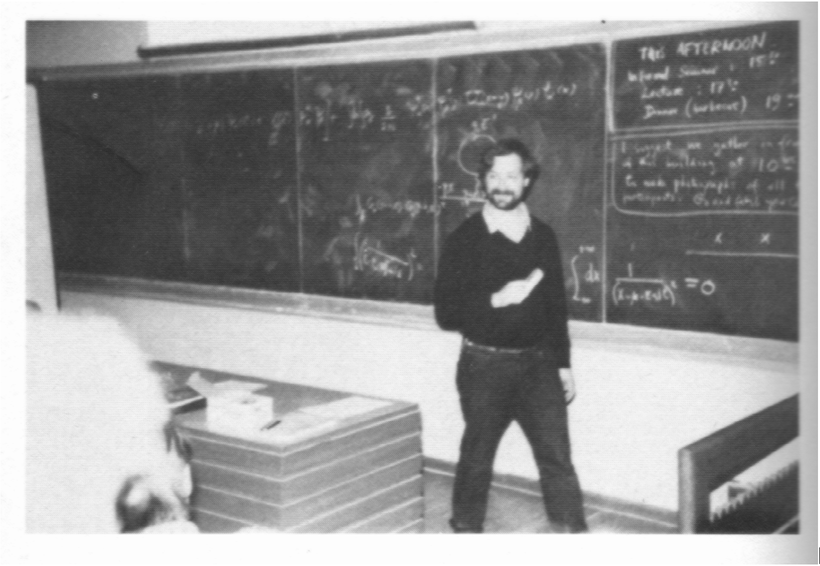

Les Houches, July 1982: Volver...Que veinte años no es nada... (C. Gardel et al, 1930)

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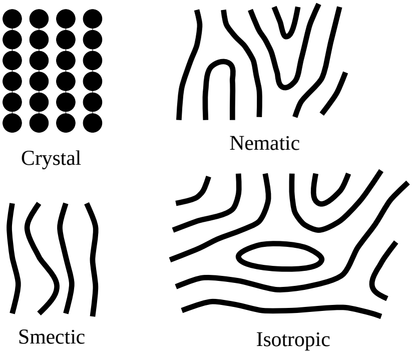
- Analogues in lipid bilayers intercalated with DNA (Lubensky et al, 2000)

Soft Quantum Matter

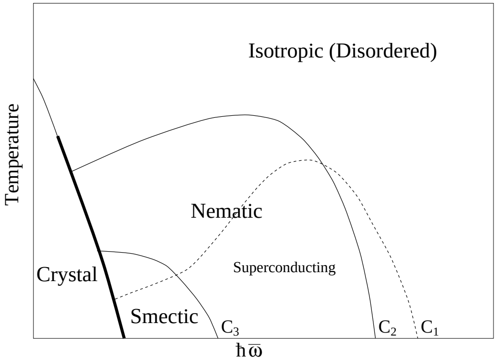
or

Quantum Soft Matter

Electron Liquid Crystal Phases

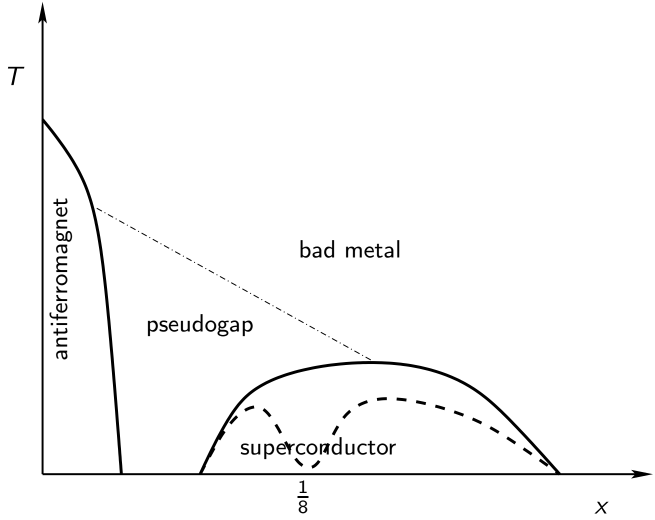


Schematic Phase Diagram of Doped Mott Insulators



$\hbar\bar{\omega}$ measures transverse zero-point stripe fluctuations of the stripes.
Systems with "large" coupling to lattice displacements (*e. g.* manganites) are "more classical" than systems with "primarily" electronic correlations (e. g. cuprates); nickelates lie in-between.

Phase Diagram of the High T_c Superconductors



Full lines: phase boundaries for the antiferromagnetic and superconducting phases.
Broken line: phase boundary for a system with static stripe order and a "1/8 anomaly"
Dotted line: crossover between the bad metal and pseudogap regimes

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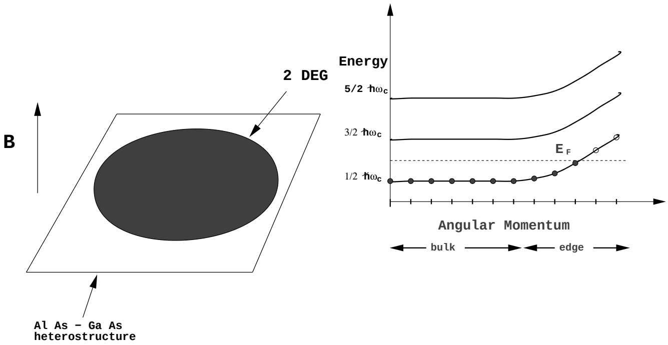

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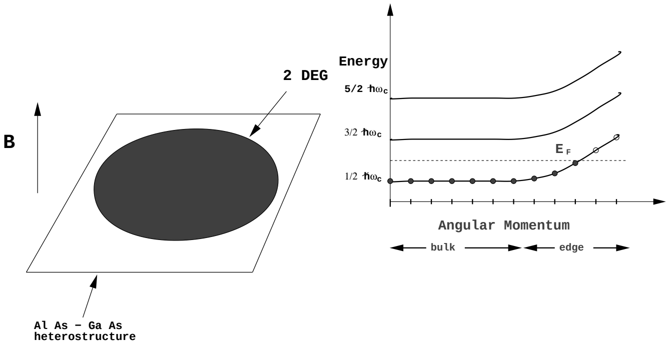

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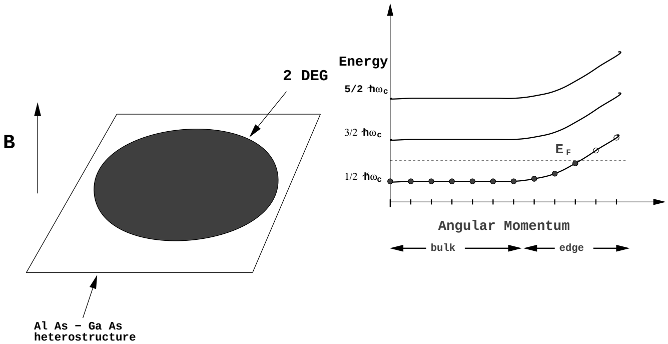

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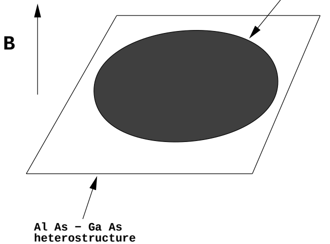

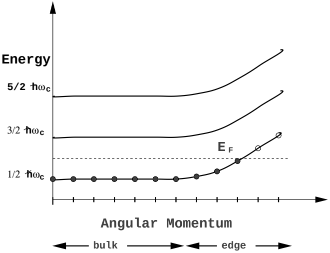

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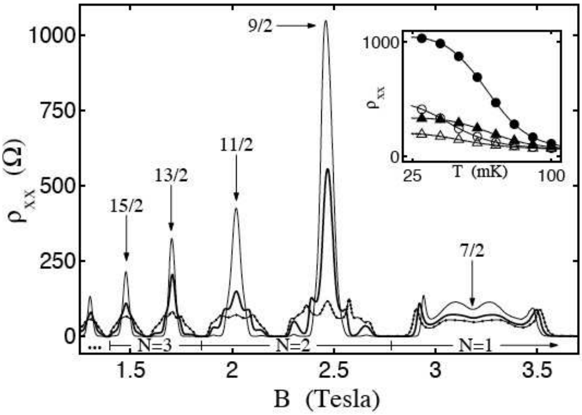

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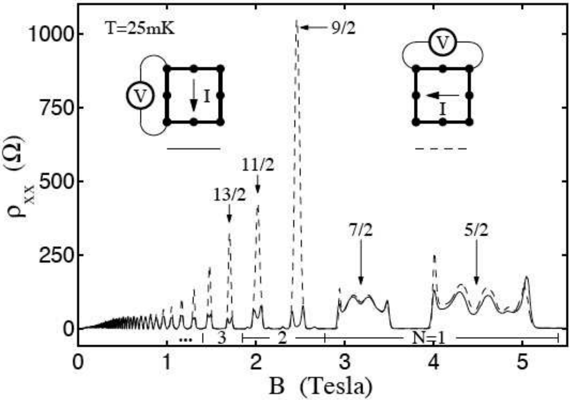

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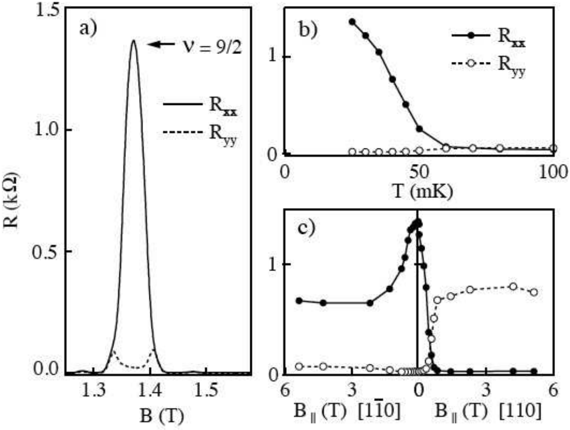

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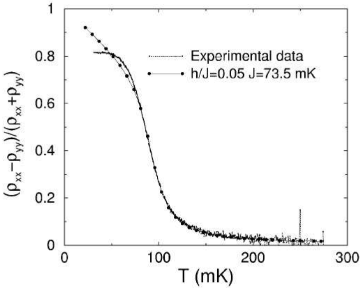
Transport Anisotropy in the 2DEG

The 2DEG behaves like a Nematic fluid!



Classical Monte Carlo simulation of a classical 2D XY model for nematic order with coupling J and external field h, on a 100×100 lattice
Fit of the order parameter to the data of M. Lilly and coworkers, at $\nu = 9/2$ (after deconvoluting the effects of the geometry.)
Best fit: $J = 73mK$ and $h = 0.05J = 3.5mK$ and $T_c = 65mK$.

E. Fradkin, S. A. Kivelson, E. Manousakis and K. Nho, Phys. Rev. Lett. 84, 1982 (2000).

K. B. Cooper *et al.*, Phys. Rev. B 65, 241313 (2002)

Transport Anisotropy in $Sr_3Ru_2O_7$ in magnetic fields

- $Sr_3Ru_2O_7$ is a strongly correlated quasi-2D bilayer oxide

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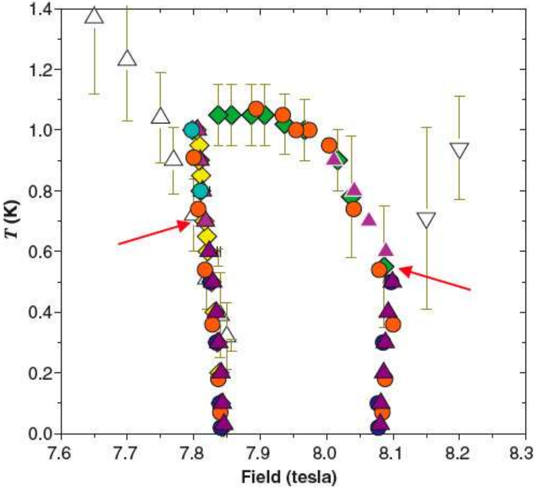
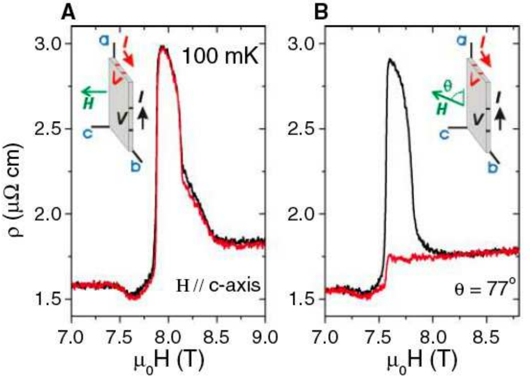


Phase diagram of $Sr_3Ru_2O_7$ in the temperature-magnetic field plane. (from Grigera et al (2004).

Transport Anisotropy in $Sr_3Ru_2O_7$ in magnetic fields



R. Borzi et al (2007)

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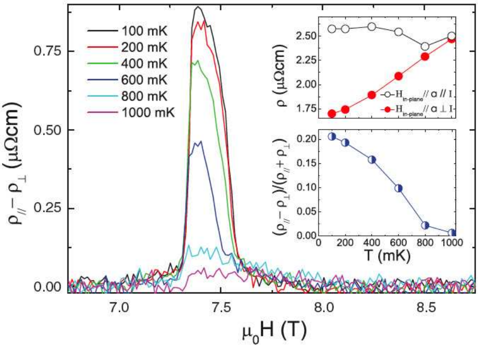
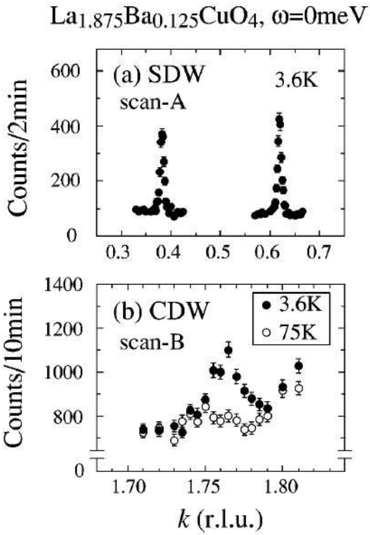


R. Borzi et al (2007)

Charge and Spin Order in the Cuprate Superconductors

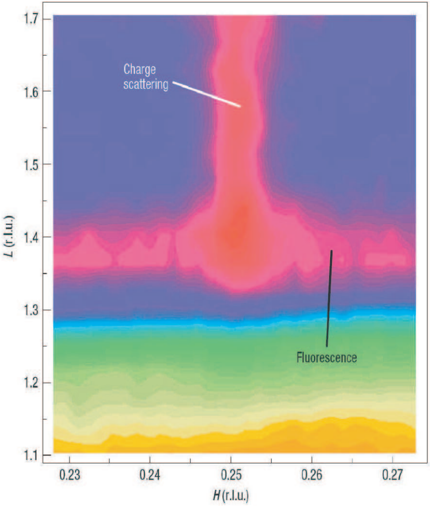
- Stripe charge order in underdoped high temperature superconductors($La_{2-x}Sr_xCuO_4$, $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$and $YBa_2Cu_3O_{6+x}$) (Tranquada, Ando, Mook, Keimer)
- Coexistence of *fluctuating* stripe charge order and superconductivity in $La_{2-x}Sr_xCuO_4$and $YBa_2Cu_3O_{6+x}$(Mook, Tranquada) and nematic order (Keimer).
- Dynamical layer decoupling in stripe ordered $La_{2-x}Ba_xCuO_4$ and in $La_{2-x}Sr_xCuO_4$ at finite fields (transport, (Tranquada et al (2007)), Josephson resonance (Basov et al (2009)))
- Induced charge order in the SC phase in vortex halos in $La_{2-x}Sr_xCuO_4$ and underdoped $YBa_2Cu_3O_{6+x}$ (neutrons: B. Lake, Keimer; STM: Davis)
- STM Experiments: short range stripe order (on scales long compared to ξ_0), possible broken rotational symmetry ($Bi_2Sr_2CaCu_2O_{8+\delta}$) (Kapitulnik, Davis, Yazdani)
- Transport experiments give evidence for charge domain switching in $YBa_2Cu_3O_{6+x}$wires (Van Harlingen/Weissmann)

Charge and Spin Order in the Cuprate Superconductors



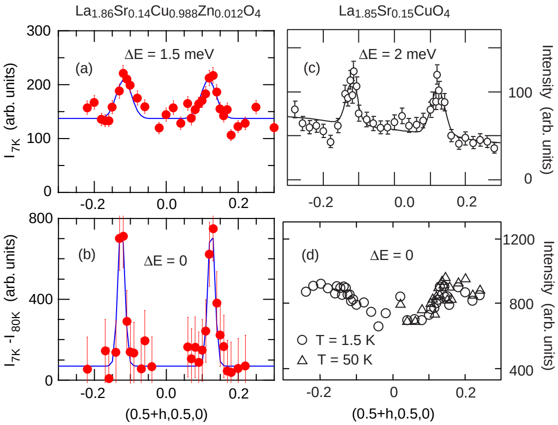
Static spin stripe order in $La_{2-x}Ba_xCuO_4$ near $x = 1/8$ in neutron scattering (Fujita et al (2004))

Charge and Spin Order in the Cuprate Superconductors



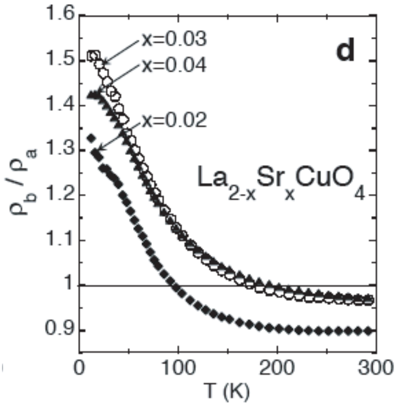
Static charge stripe order in $La_{2-x}Ba_xCuO_4$ near $x = 1/8$ in resonant X-ray scattering (Abbamonte et. al.(2005))

Induced stripe order in $La_{2-x}Sr_xCuO_4$ by *Zn* impurities

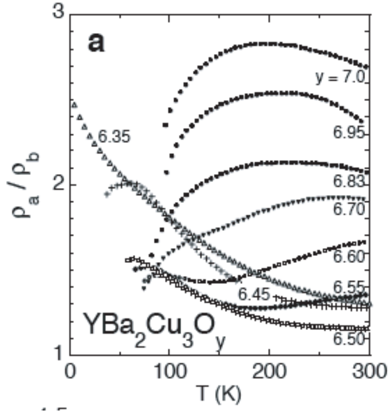


Magnetic neutron scattering with and without *Zn* (Kivelson et al (2003))

Electron Nematic Order in High Temperature Superconductors



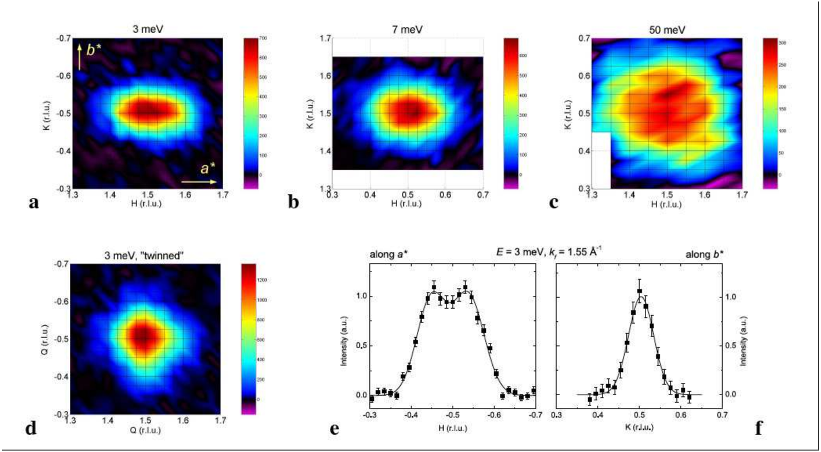
(i)



(j)

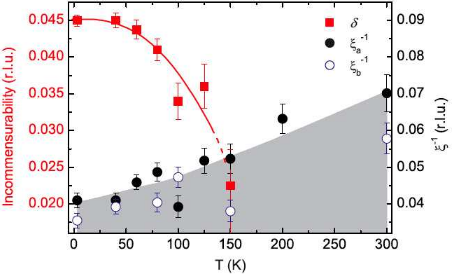
Temperature-dependent transport anisotropy in underdoped $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_{6+x}$; Ando et al (2002)

Charge Nematic Order in underdoped $YBa_2Cu_3O_{6+x}$ ($y = 6.45$)



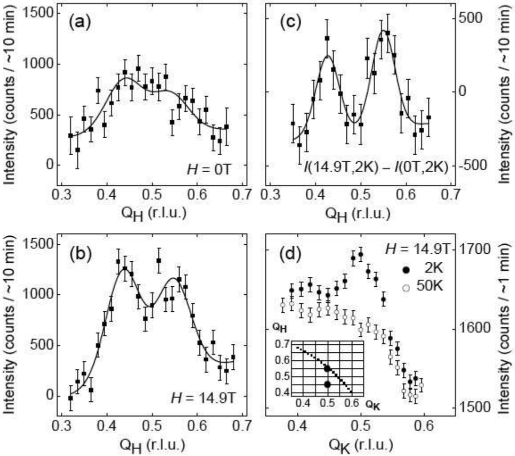
Intensity maps of the spin-excitation spectrum at 3, 7,and 50 meV, respectively. Colormap of the intensity at 3 meV, as it would be observed in a crystal consisting of two perpendicular twin domains with equal population. Scans along a^* and b^* through $\mathbf{Q}_{AF}$. (from Hinkov et al (2007).

Charge Nematic Order in underdoped $YBa_2Cu_3O_{6+x}$ ($y = 6.45$)



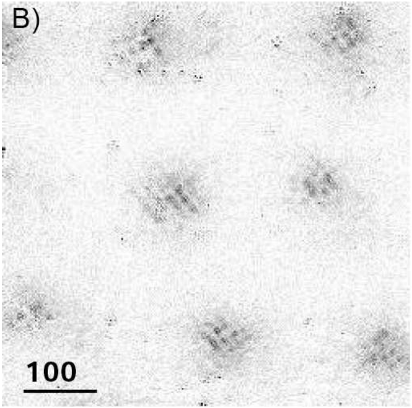
a) Incommensurability δ (red symbols), half-width-at-half-maximum of the incommensurate peaks along a^* (ξ_a^{-1}, black symbols) and along b^* (ξ_b^{-1}, open blue symbols) in reciprocal lattice units. (from Hinkov et al 2007)).

Static stripe order in underdoped $YBa_2Cu_3O_{6+x}$ at finite fields



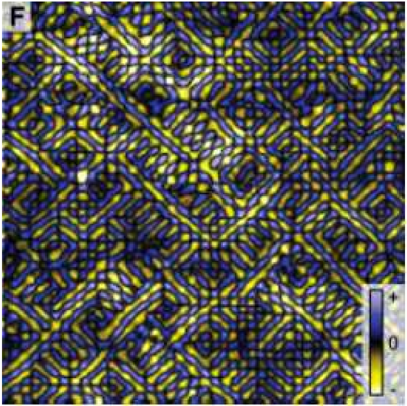
Hinkov et al (2008)

Charge Order induced inside a SC vortex "halo"

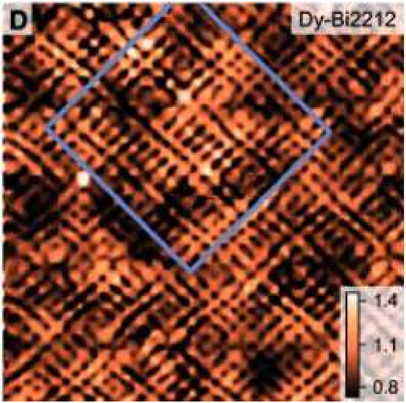


Induced charge order in the SC phase in vortex halos: neutrons in $La_{2-x}Sr_xCuO_4$ (B. Lake *et al*, 2002), STM in optimally doped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (S. Davis *et al*, 2004)

STM: Short range stripe order in Dy-$Bi_2Sr_2CaCu_2O_{8+\delta}$



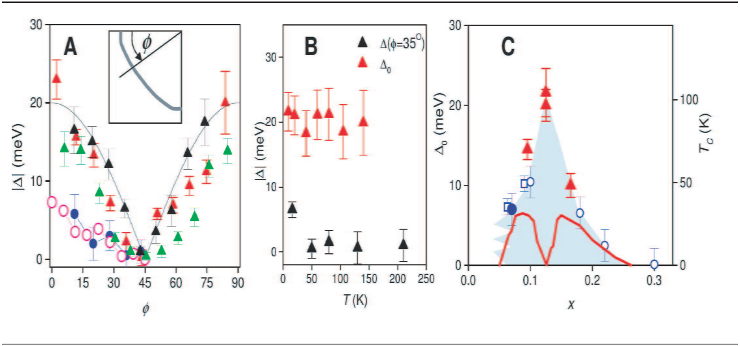
(k) Left



(l) Right

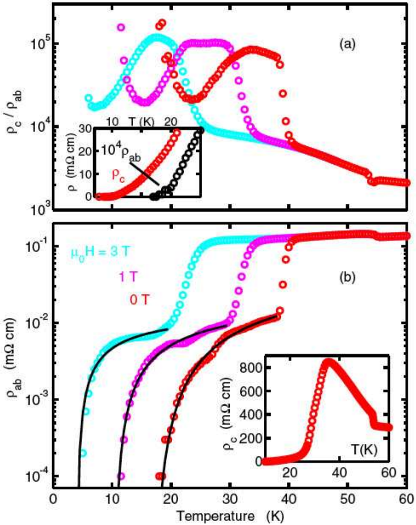
Left: STM R-maps in Dy-$Bi_2Sr_2CaCu_2O_{8+\delta}$ at high bias:
$R(\vec{r}, 150\ mV) = I(\vec{r}, +150 mV)/I(\vec{r}, -150\ mV)$
Right: Short range nematic order with $\gg \xi_0$; Kohsaka et al (2007)

Optimal Degree of Inhomogeneity in $La_{2-x}Ba_xCuO_4$



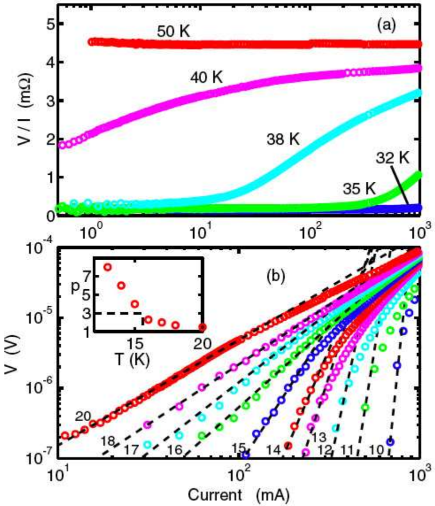
ARPES in $La_{2-x}Ba_xCuO_4$: the antinodal (pairing) gap is *largest*, even though T_c is *lowest*, near $x = \frac{1}{8}$; T. Valla et al (2006), ZX Shen et al (2008)

Dynamical Layer Decoupling in $La_{2-x}Ba_xCuO_4$ near $x = 1/8$



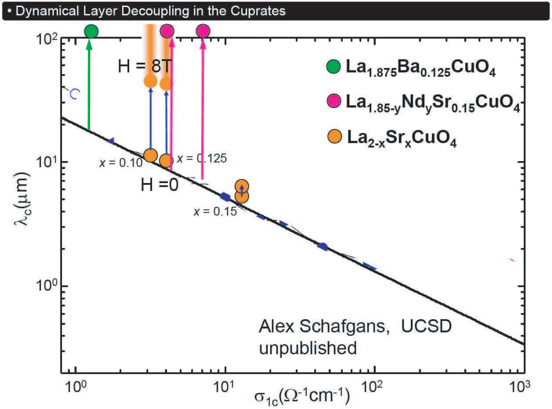
Dynamical layer decoupling in transport (Li et al (2008))

Dynamical Layer Decoupling in $La_{2-x}Ba_xCuO_4$ near $x = 1/8$



Kosterlitz-Thouless resistive transition (Li et al (2008)

Dynamical Layer Decoupling in $La_{2-x}Sr_xCuO_4$ in a magnetic field



Layer decoupling seen in Josephson resonance spectroscopy: c-axis penetration depth λ_c vs c-axis conductivity σ_{1c} (Basov et al, 2009)

Theory of the Nematic Fermi Fluid

V. Oganesyan, S. A. Kivelson, and E. Fradkin (2001)

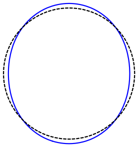
- The nematic order parameter for two-dimensional Fermi fluid is the 2×2 symmetric traceless tensor

$$\hat{\mathbf{Q}}(x) \equiv -\frac{1}{k_F^2}\Psi^\dagger(\vec{r}) \begin{pmatrix} \partial_x^2 - \partial_y^2 & 2\partial_x\partial_y \\ 2\partial_x\partial_y & \partial_y^2 - \partial_x^2 \end{pmatrix} \Psi(\vec{r}),$$

- It can also be represented by a complex valued field $\mathcal{Q}$ whose expectation value is the nematic phase is

$$\langle \mathcal{Q} \rangle \equiv \langle \Psi^\dagger \left(\partial_x + i\partial_y\right)^2 \Psi \rangle = |\mathcal{Q}|\, e^{2i\theta} = \mathcal{Q}_{11} + i\mathcal{Q}_{12} \neq 0$$

- $\mathcal{Q}$ carries angular momentum $\ell = 2$.
- $\langle \mathcal{Q} \rangle \neq 0 \Rightarrow$ the Fermi surface spontaneously distorts and becomes an ellipse with eccentricity $\propto Q \Rightarrow$ This state breaks rotational invariance mod π



Theory of the Nematic Fermi Fluid

A Fermi liquid model: "Landau on Landau"

- $$H = \int d\vec{r}\, \Psi^\dagger(\vec{r})\epsilon(\vec{\nabla})\Psi(\vec{r}) + \frac{1}{4}\int d\vec{r} \int d\vec{r}' F_2(\vec{r}-\vec{r}')\mathrm{Tr}[\hat{\mathbf{Q}}(\vec{r})\hat{\mathbf{Q}}(\vec{r}')]$$

  $\epsilon(\vec{k}) = v_F q[1 + a(\frac{q}{k_F})^2]$, $q \equiv |\vec{k}| - k_F$
  $F_2(\vec{r}) = (2\pi)^{-2}\int d\vec{k} e^{i\vec{q}\cdot\vec{r}} F_2/[1+\kappa F_2 q^2]$
  F_2 is a Landau parameter.
- Landau energy density functional:

  $$\mathcal{V}[\mathbf{Q}] = E(\mathbf{Q}) - \frac{\tilde{\kappa}}{4}\mathrm{Tr}[\mathbf{QDQ}] - \frac{\tilde{\kappa}'}{4}\mathrm{Tr}[\mathbf{Q}^2\mathbf{DQ}] + \ldots$$

  $$E(\mathbf{Q}) = E(\mathbf{0}) + \frac{A}{4}\mathrm{Tr}[\mathbf{Q}^2] + \frac{B}{8}\mathrm{Tr}[\mathbf{Q}^4] + \ldots$$

  where $A = \frac{1}{2N_F} + F_2$, N_F is the density of states at the Fermi surface, $B = \frac{3aN_F|F_2|^3}{8E_F^2}$, and $E_F \equiv v_F k_F$ is the Fermi energy.
- If $A < 0 \Rightarrow$ nematic phase

Theory of the Nematic Fermi Fluid

This model has two phases:

- an isotropic Fermi liquid phase
- a nematic non-Fermi liquid phase

separated by a quantum critical point at $2N_F F_2 = -1$

Quantum Critical Behavior

- Parametrize the distance to the Pomeranchuk QCP by $\delta = -1/2 - 1/N_F F_2$ and define $s = \omega / q v_F$
- The transverse collective nematic modes have Landau damping at the QCP. Their effective action has a kernel

$$\kappa q^2 + \delta - i \frac{\omega}{q v_F}$$

- The dynamic critical exponent is $z = 3$.

Theory of the Nematic Fermi Fluid

Physics of the Nematic Phase:

- Transverse Goldstone boson which is generically overdamped except for $\phi = 0, \pm\pi/4, \pm\pi/2$ (symmetry directions) where it is underdamped
- Anisotropic (Drude) Transport

$$\frac{\rho_{xx} - \rho_{yy}}{\rho_{xx} + \rho_{yy}} = \frac{1}{2}\frac{m_y - m_x}{m_y + m_x} = \frac{\mathrm{Re}\ Q}{E_F} + \mathcal{O}(Q^3)$$

- Quasiparticle scattering rate (one loop);
  - In general

$$\Sigma''(\epsilon, \vec{k}) = \frac{\pi}{\sqrt{3}}\frac{(\kappa k_F^2)^{1/3}}{\kappa N_F}\left|\frac{k_x k_y}{k_F^2}\right|^{4/3}\left|\frac{\epsilon}{2 v_F k_F}\right|^{2/3} + \ldots$$

  - Along a symmetry direction:

$$\Sigma''(\epsilon) = \frac{\pi}{3 N_F \kappa}\frac{1}{(\kappa k_F^2)^{1/4}}\left|\frac{\epsilon}{v_F k_F}\right|^{3/2} + \ldots$$

- The Nematic Phase is a non-Fermi Liquid!

Local Quantum Criticality at the Nematic QCP

- Since $\Sigma''(\omega) \gg \Sigma'(\omega)$ (as $\omega \to 0$), we need to asses the validity of these results as they signal a *failure of perturbation theory*
- We use higher dimensional bosonization as a non-perturbative tool (Haldane 1993, Castro Neto and Fradkin 1993, Houghton and Marston 1993)
- Higher dimensional bosonization reproduces the collective modes found in Hartree-Fock+ RPA and is consistent with the Hertz-Millis analysis of quantum criticality: $d_{\rm eff} = d + z = 5$. (Lawler et al, 2006)
- The fermion propagator takes the form

$$G_F(x,t) = G_0(x,t) Z(x,t)$$

Local Quantum Criticality at the Nematic QCP

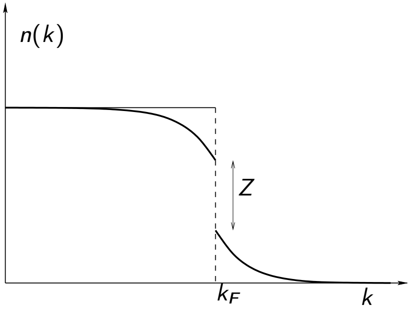
Lawler et al , 2006

- At the Nematic-FL QCP:

$$G_F(x,0) = G_0(x,0)\, e^{-\text{const.}\,|x|^{1/3}},$$

$$G_F(0,t) = G_0(0,t)\, e^{-\text{const}'.\,|t|^{-2/3}\ln t},$$

- Quasiparticle residue: $Z = \lim_{x\to\infty} Z(x,0) = 0!$
- DOS: $N(\omega) = N(0)\left(1 - \text{const}'.|\omega|^{2/3}\ln\omega\right)$



Local Quantum Criticality at the Nematic QCP

- Behavior near the QCP: for $T = 0$ and $\delta \ll 1$ (FL side), $Z \propto e^{-\mathrm{const.}/\sqrt{\delta}}$
- At the QCP $((\delta = 0),\ T_F \gg T \gg T_\kappa)$

$$Z(x, 0) \propto e^{-\mathrm{const.}\ Tx^2 \ln(L/x)} \to 0$$

  but, $Z(0, t)$ is *finite* as $L \to \infty$!
- "Local quantum criticality"
- Irrelevant quartic interactions of strength u lead to a renormalization that smears the QCP at $T > 0$ (Millis 1993)

$$\delta \to \delta(T) = -uT \ln\left(uT^{1/3}\right)$$

- $$Z(x, 0) \propto e^{-\mathrm{const.}\ Tx^2 \ln(\xi/x)}, \qquad \text{where}\ \ \xi = \delta(T)^{-1/2}$$

Generalizations: Charge Order in Higher Angular Momentum Channels

- Higher angular momentum particle-hole condensates

$$\langle \mathcal{Q}_\ell \rangle = \langle \Psi^\dagger \left(\partial_x + i\partial_y\right)^\ell \rangle$$

- For ℓ *odd*: breaks rotational invariance (mod $2\pi/\ell$). It also breaks parity $\mathcal{P}$ and time reversal $\mathcal{T}$ but $\mathcal{PT}$ is invariant; *e.g.* For $\ell = 3$ ("Varma loop state")
- ℓ *even*: Hexatic ($\ell = 6$), etc.

Nematic Order in the Triplet Channel

Wu, Sun, Fradkin and Zhang (2007)

- Order Parameters in the Spin Triplet Channel $(\alpha, \beta = \uparrow, \downarrow)$

$$\mathcal{Q}_\ell^a(r) = \langle \Psi_\alpha^\dagger(r) \sigma_{\alpha\beta}^a \left(\partial_x + i\partial_y\right)^\ell \Psi_\beta(r) \rangle \equiv n_1^a + i n_2^a$$

- $\ell \neq 0 \Rightarrow$ Broken rotational invariance in space and in spin space; particle-hole condensate analog of He_3A and He_3B
- Time Reversal: $\mathcal{T} \mathcal{Q}_\ell^a \mathcal{T}^{-1} = (-1)^{\ell+1} \mathcal{Q}_\ell^a$
- Parity: $\mathcal{P} \mathcal{Q}_\ell^a \mathcal{P}^{-1} = (-1)^\ell \mathcal{Q}_\ell^a$
- $\mathcal{Q}_\ell^a$ rotates under an $SO_{\rm spin}(3)$ transformation, and $\mathcal{Q}_\ell^a \to \mathcal{Q}_\ell^a e^{i\ell\theta}$ under a rotation in space by θ
- $\mathcal{Q}_\ell^a$ is invariant under a rotation by π/ℓ and a spin flip

Nematic Order in the Triplet Channel

Landau theory

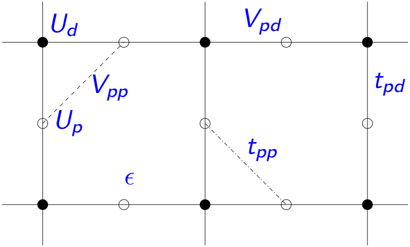
- Free Energy

$$F[n] = r(|\vec{n}_1|^2 + |\vec{n}_2|^2) + v_1(|\vec{n}_1|^2 + |\vec{n}_2|^2)^2 + v_2|\vec{n}_1 \times \vec{n}_2|^2$$

- Pomeranchuk: $r < 0$, $(F_\ell^A < -2,\ \ell \geq 1)$
  - $v_2 > 0, \Rightarrow \vec{n}_1 \times \vec{n}_2 = 0$ ("α" phase)
  - $v_2 > 0, \Rightarrow \vec{n}_1 \cdot \vec{n}_2 = 0$ and $|\vec{n}_1| = |\vec{n}_2|$, ("$\beta$" phase)
- $\ell = 2$ α phase: "nematic-spin-nematic"; in this phase the spin up and spin down FS have an $\ell = 2$ nematic distortion but are rotated by $\pi/2$
- In the β phases there are two isotropic FS bur spin is not a good quantum number: there is an effective spin orbit interaction!
- Define a d vector: $\vec{d}(k) = (\cos(\ell\theta_k), \sin(\ell\theta_k), 0)$; In the β phases it winds about the undistorted FS: for $\ell = 1$, $w = 1$ "Rashba", $w = -1$ "Dresselhaus"

Nematic States in the Strongly Coupled Emery Model of a CuO plane

S.A. Kivelson, E.Fradkin and T. Geballe (2004)



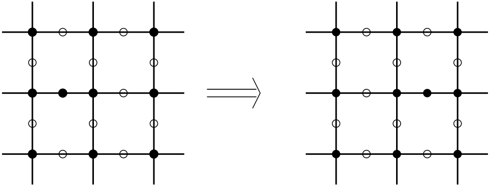
Energetics of the 2D $Cu - O$ model in the strong coupling limit:
$t_{pd}/U_p, t_{pd}/U_d, t_{pd}/V_{pd}, t_{pd}/V_{pp} \to 0$
$U_d > U_p \gg V_{pd} > V_{pp}$ and $t_{pp}/t_{pd} \to 0$
as a function of hole doping $x > 0$ ($x = 0 \Leftrightarrow$ half-filling)

Energy to add one hole: $\mu \equiv 2V_{pd} + \epsilon$
Energy of two holes on nearby O sites: $\mu + V_{pp} + \epsilon$

Nematic States in the Strongly Coupled Emery Model of a CuO plane

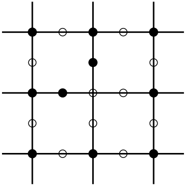
Effective One-Dimensional Dynamics at Strong Coupling

In the strong coupling limit, and at $t_{pp} = 0$, the motion of an extra hole is strongly constrained. The following is an allowed move which takes two steps.

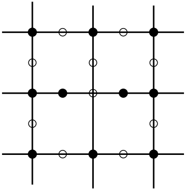


The final and initial states are degenerate, and their energy is $E_0 + \mu$

Nematic States in the Strongly Coupled Emery Model



a)



b)

a) Intermediate state for the hole to turn a corner; it has energy $E_0 + \mu + V_{pp}$
$\Rightarrow t_{\rm eff} = \frac{t_{pd}^2}{V_{pp}} \ll t_{pd}$
b) Intermediate state for the hole to continue on the same row; it has energy $E_0 + \mu + \epsilon \Rightarrow t_{\rm eff} = \frac{t_{pd}^2}{\epsilon}$

Nematic States in the Strongly Coupled Emery Model

- The ground state at $x=0$ is an antiferromagnetic insulator
- Doped holes behave like one-dimensional spinless fermions

$$H_c = -t\sum_j [c_j^\dagger c_{j+1} + h.c.] + \sum_j [\epsilon_j \hat{n}_j + V_{pd}\hat{n}_j\hat{n}_{j+1}]$$

- at $x=1$ it is a Nematic insulator
- the ground state for $x \to 0$ and $x \to 1$ is a uniform array of 1D Luttinger liquids $\Rightarrow$ it is an Ising Nematic Phase.
- This result follows since for $x \to 0$ the ground state energy is

$$E_{\mathrm{nematic}} = E(x=0) + \Delta_c\; x + W\; x^3 + O(x^5)$$

where $\Delta_c = 2V_{pd} + \epsilon + \ldots$ and $W = \pi^2\hbar^2/6m^*$, while the energy of the isotropic state is

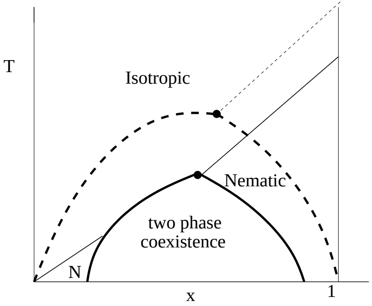
$$E_{\mathrm{isotropic}} = E(x=0) + \Delta_c\; x + (1/4)W\; x^3 + V_{\mathrm{eff}}\; x^2$$

(V_{eff}: effective coupling for holes on intersecting rows and columns)
$\Rightarrow E_{\mathrm{nematic}} < E_{\mathrm{isotropic}}$

- A similar argument holds for $x \to 1$.
- the density of mobile charge $\sim x$ but $k_F = (1-x)\pi/2$
- For $t_{pp} \neq 0$ this 1D state crosses over (most likely) to a 2D (Ising) Nematic Fermi liquid state.

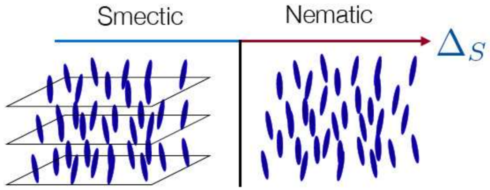
Nematic States in the Strongly Coupled Emery Model

Phase diagram in the strong coupling limit



- In the "Classical" Regime, $\epsilon/t_{pd} \to \infty$, with $U_d > \epsilon$, the doped holes are distributed on O sites at an energy cost μ per doped hole and an interaction $J = V_{pp}/4$ per neighboring holes on the O sub-lattice
- This is a classical lattice gas equivalent to a 2D classical Ising antiferromagnet with exchange J in a uniform "field" μ, and an effective magnetization (per O site) $m = 1 - x$
- The classical Ising antiferromagnet at temperature T and magnetization $m = 1 - x$ has the phase diagram of the figure.
- Quantum fluctuations lead to a similar phase diagram, except for the extra nematic phase.

The Quantum Nematic-Smectic Phase Transition



$$\boxed{S = \int d^d\vec{r}dt \left[\left|\partial_t \Phi\right|^2 - \left|(\vec{\nabla} - iQ_S\delta\vec{n})\Phi\right|^2 - \Delta_S|\Phi|^2 - u_S|\Phi|^4 \right] + S_N\left[\delta\vec{n}\right]}$$

- Q_S is ordering wave vector
- Φ is a (matter) density field
- Gauge-like coupling of director field

$\boxed{\text{But this is for insulators!}}$

The Quantum Nematic-Smectic Phase Transition

Coupling to electrons

Electron quadrupole density (g_N):

$$Q(\vec{r}, t) = k_F^{-2} \psi^\dagger (\partial_x + i\partial_y)^2 \psi$$

Electron charge density near Q_S (g_S):

$$n(\vec{q}, \omega) = \int \frac{d\vec{k}\, d\Omega}{(2\pi)^{d+1}} \psi^\dagger(\vec{k} + \vec{Q}_S + \vec{q}, \Omega + \omega)\psi(\vec{k}, \Omega)$$

Linear coupling allowed:

$$S_F = \int d\vec{r}dt \left[g_N Q^\dagger N + g_S n^\dagger \Phi + h.c. \right]$$

Phenomenological metallic theory

The Quantum Nematic-Smectic Phase Transition

The smectic-nematic quantum critical point

Gauge-like couping for metallic case is irrelevant

So phase transition can be continuous!

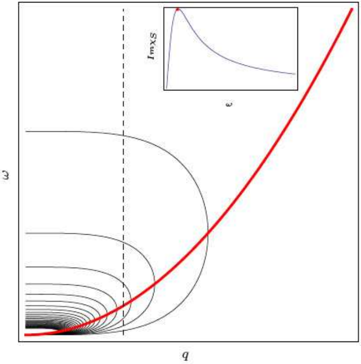
Over damped smectic mode

$$\omega_{\vec{q}} = \frac{q_x^2}{2m_x} + \frac{q_y^2}{2m_y} \quad \text{(z=2)}$$

Over damped nematic mode

$$\omega_{\vec{q}} \propto q^3 \quad \text{(z=3)}$$

The Quantum Nematic-Smectic Phase Transition



Stripe susceptibility

$$\chi^S = \frac{Z}{i\omega - q_x^2/2m_x - q_y^2/2m_y}$$

Can be measured with inelastic X-ray scattering

The Quantum Nematic-Smectic Phase Transition

The metallic smectic phase

Smectic order parameter can "eat" the nematic order parameter $\Phi = \Phi_0 e^{iq_0 u}$

$$\boxed{S_{smectic} = \int \frac{d^2q d\omega}{(2\pi)^3} \left(\alpha\omega^2 + i|\omega|\sqrt{q_y^2 + \lambda^2 q_x^4} - q_y^2 - \lambda^2 q_x^4 \right) |u|^2}$$

Above upper critical dimension

$$[q_x] = 1, \quad [q_y] = 2, \quad [\omega] = 2$$

No long lived quasi-particles

$$Im\Sigma \propto \log \omega$$

The Quantum Nematic-Smectic Phase Transition

Different possibilities

	Smectic Mode at the Electronic Nematic-Smectic QCP			
	$Q_S < 2k_F$	$Q_S = 2k_F$ incommensurate	$Q_S = 2k_F$ commensurate	inflection point
Anisotropic Scaling $[q_x] : [q_y] : [\omega]$	$1:1:2$	$1:2:3$	$1:2:2$	$1:3:3$
Non-analyticity		$\Phi^{5/2}$	$\Phi^{5/2}$	$\Phi^{9/4}$
Gaussian Fixed Point	Stable	Unstable / First Order	Stable	Stable
$\Sigma''(k_F, \omega)$	$\|\omega\|^{1/2}$	?	$\|\omega\|$	$\|\omega\|^{13/12}$

	Smectic	
	continuous rotational symmetry	discrete rotational symmetry
Anisotropic Scaling $[q_x] : [q_y] : [\omega]$	$1:2:2$	$1:1:1$
Non-analyticity		
Gaussian Fixed Point	Stable	Stable
$\Sigma''(k_F, \omega)$	$\log \|\omega\|$ or $\|\omega\|^{-1/2}$	const.

Stripe Phases and the Mechanism of high temperature superconductivity in Strongly Correlated Systems

- Since the discovery of high temperature superconductivity it has been clear that
  - High Temperature Superconductors are never normal metals and don't have well defined quasiparticles in the "normal state" (linear resistivity, ARPES)
  - the "parent compounds" are strongly correlated Mott insulators
  - repulsive interactions dominate
  - the quasiparticles are an 'emergent' low-energy property of the superconducting state
  - whatever "the mechanism" is has to account for these facts

Stripe Phases and the Mechanism of high temperature superconductivity in Strongly Correlated Systems

Problem

BCS is so successful in conventional metals that the term mechanism naturally evokes the idea of a weak coupling instability with (write here your favorite boson) mediating an attractive interaction between well defined quasiparticles. The basic assumptions of BCS theory are not satisfied in these systems.

Stripe Phases and the Mechanism of HTSC

Superconductivity in a Doped Mott Insulator

or How To Get Pairing from Repulsive Interactions

- Universal assumption: 2D Hubbard-like models should contain the essential physics
- "RVB" mechanism:
  - Mott insulator: spins are bound in singlet valence bonds; it is a strongly correlated spin liquid, essentially a pre-paired insulating state
  - spin-charge separation in the doped state leads to high temperature superconductivity

Stripe Phases and the Mechanism of HTSC

Problems

- there is no real evidence that the simple 2D Hubbard model favors superconductivity (let alone high temperature superconductivity)
- all evidence indicates that if anything it wants to be an insulator and to phase separate (finite size diagonalizations, various Monte Carlo simulations)
- strong tendency for the ground states to be inhomogeneous and possibly anisotropic
- no evidence (yet) for a spin liquid in 2D Hubbard-type models

Stripe Phases and the Mechanism of HTSC

Why an Inhomogeneous State is Good for high T_c SC

- "Inhomogeneity induced pairing" mechanism: "pairing" from strong repulsive interactions.
- Repulsive interactions lead to local superconductivity on 'mesoscale structures'
- The strength of this pairing tendency decreases as the size of the structures increases above an optimal size
- The physics responsible for the pairing within a structure $\Rightarrow$ Coulomb frustrated phase separation $\Rightarrow$ mesoscale electronic structures

Stripe Phases and the Mechanism of HTSC

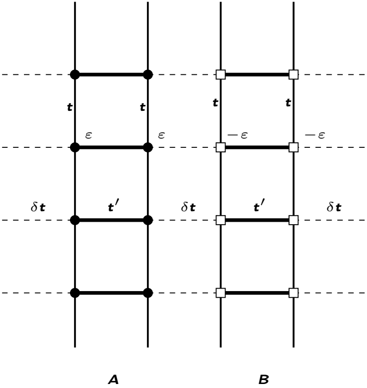
Pairing and Coherence

- Strong local pairing does not guarantee a large critical temperature
  - In an isolated system, the phase ordering (condensation) temperature is suppressed by phase fluctuations, often to $T = 0$
  - The highest possible T_c is obtained with an intermediate degree of inhomogeneity
  - The optimal T_c always occurs at a point of crossover from a pairing dominated regime when the degree of inhomogeneity is suboptimal, to a phase ordering regime with a pseudo-gap when the system is too 'granular'

Stripe Phases and the Mechanism of HTSC

A Cartoon of the Strongly Correlated Stripe Phase

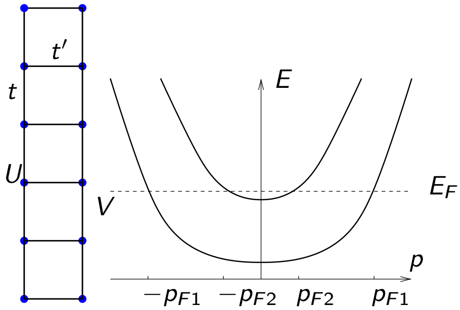
$$H = - \sum_{<\vec{r},\vec{r}'>,\sigma} t_{\vec{r},\vec{r}'} \left[c^\dagger_{\vec{r},\sigma} c_{\vec{r}',\sigma} + \mathrm{h.c.} \right] + \sum_{\vec{r},\sigma} \left[\epsilon_{\vec{r}}\, c^\dagger_{\vec{r},\sigma} c_{\vec{r},\sigma} + \frac{U}{2} c^\dagger_{\vec{r},\sigma} c^\dagger_{\vec{r},-\sigma} c_{\vec{r},-\sigma} c_{\vec{r},\sigma} \right]$$



Arrigoni, Fradkin and Kivelson (2004)

Stripe Phases and the Mechanism of HTSC

Physics of the 2-leg ladder



- $U = V = 0$: two bands with different Fermi wave vectors, $p_{F1} \neq p_{F2}$
- The only allowed processes involve an *even* number of electrons
- Coupling of CDW fluctuations with $Q_1 = 2p_{F1} \neq Q_2 = 2p_{F2}$ is suppressed

Stripe Phases and the Mechanism of HTSC

Why is there a Spin Gap

- Scattering of electron pairs with zero center of mass momentum from one system to the other is peturbatively relevant
- The electrons can gain zero-point energy by delocalizing between the two bands.
- The electrons need to pair, which may cost some energy.
- When the energy gained by delocalizing between the two bands exceeds the energy cost of pairing, the system is driven to a spin-gap phase.

Stripe Phases and the Mechanism of HTSC

What is it known about the 2-leg ladder

- $x = 0$: unique fully gapped ground state ("$C0S0$"); for $U \gg t$, $\Delta_s \sim J/2$
- For $0 < x < x_c \sim 0.3$, Luther-Emery liquid: no charge gap and large spin gap ("$C1S0$"); spin gap $\Delta_s \downarrow$ as $x \uparrow$, and $\Delta_s \to 0$ as $x \to x_c$
- Effective Hamiltonian for the charge degrees of freedom

$$H = \int dy \frac{v_c}{2} \left[K \left(\partial_y \theta\right)^2 + \frac{1}{K} \left(\partial_x \phi\right)^2 \right] + \dots$$

ϕ: CDW phase field; θ: SC phase field; $[\phi(y'), \partial_y \theta(y)] = i\delta(y - y')$

- x-dependence of Δ_s, K, v_c, and μ depends on t'/t and U/t
- $\dots$ represent cosine potentials: Mott gap Δ_M at $x = 0$
- $K \to 2$ as $x \to 0$; $K \sim 1$ for $x \sim 0.1$, and $K \sim 1/2$ for $x \sim x_c$
- $\chi_{\rm SC} \sim \Delta_s / T^{2-K^{-1}}$ $\qquad$ $\chi_{\rm CDW} \sim \Delta_s / T^{2-K}$
- $\chi_{\rm CDW}(T) \to \infty$ and $\chi_{\rm SC}(T) \to \infty$ for $0 < x < x_c$
- For $x \lesssim 0.1$, $\chi_{\rm SC} \gg \chi_{\rm CDW}$!

Stripe Phases and the Mechanism of HTSC

Effects of Inter-ladder Couplings

- Luther-Emery phase: spin gap and no single particle tunneling
- Second order processes in δt:
  - marginal (and small) forward scattering inter-ladder interactions
  - possibly relevant couplings: Josephson and CDW
- Relevant Perturbations

$$H' = -\sum_J \int dy \left[\mathcal{J} \cos\left(\sqrt{2\pi}\Delta\theta_J)\right) + \mathcal{V} \cos\left(\Delta P_J y + \sqrt{2\pi}\Delta\phi_J)\right)\right]$$

  J: ladder index; $P_J = 2\pi x_J$, $\Delta\phi_J = \phi_{J+1} - \phi_J$, etc.

- $\mathcal{J}$ and $\mathcal{V}$ are effective couplings which must be computed from microscopics; estimate: $\mathcal{J} \approx \mathcal{V} \propto (\delta t)^2/J$

Stripe Phases and the Mechanism of HTSC

Period 2 works for $x \ll 1$

- If all the ladders are equivalent: period 2 stripe (columnar) state (Sachdev and Vojta)
- Isolated ladder: $T_c = 0$
- $\mathcal{J} \neq 0$ and $\mathcal{V} \neq 0$, $T_C > 0$
- $x \lesssim 0.1$: CDW couplings are irrelevant $(1 < K < 2) \Rightarrow$ Inter-ladder Josephson coupling leads to a SC state in a small x with low T_c.

$$2\mathcal{J}\chi_{\mathrm{SC}}(T_c) = 1$$

- $T_c \propto \delta t \; x$
- For larger x, $K < 1$ and χ_{CDW} is more strongly divergent than χ_{SC}
- CDW couplings become more relevant $\Rightarrow$ Insulating, incommensurate CDW state with ordering wave number $P = 2\pi x$.

Stripe Phases and the Mechanism of HTSC

Period 4 works!

- Alternating array of inequivalent A and B type ladders in the LE regime
- SC T_c:
$$(2\mathcal{J})^2 \chi^A_{\mathrm{SC}}(T_c)\chi^B_{\mathrm{SC}}(T_c) = 1$$
- CDW T_c:
$$(2\mathcal{V})^2 \chi^A_{\mathrm{CDW}}(P, T_c)\chi^B_{\mathrm{CDW}}(P, T_c) = 1$$
- 2D CDW order is greatly suppressed due to the mismatch between ordering vectors, P_A and P_B, on neighboring ladders

Stripe Phases and the Mechanism of HTSC

Period 4 works!

For inequivalent ladders SC beats CDW if

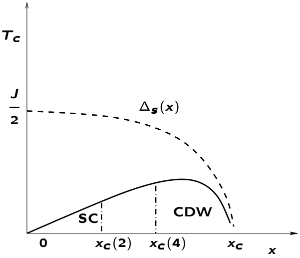
- $$2 > K_A^{-1} + K_B^{-1} - K_A; \quad 2 > K_A^{-1} + K_B^{-1} - K_B$$
- $$T_c \sim \Delta_s \left(\frac{\mathcal{J}}{\widetilde{W}}\right)^{\alpha}; \ \alpha = \frac{2K_A K_B}{[4K_A K_B - K_A - K_B]}$$
- $\mathcal{J} \sim \delta t^2/J$ and $\widetilde{W} \sim J$; T_c is (power law) small for small $\mathcal{J}$! ($\alpha \sim 1$).

Stripe Phases and the Mechanism of HTSC

How reliable are these estimates?

- This is a mean-field estimate for T_c and it is an upper bound to the actual T_c.
- T_c should be suppressed by phase fluctuations by up to a factor of 2.
- Indeed, perturbative RG studies for small $\mathcal{J}$ yield the same power law dependence. This result is asymptotically exact for $\mathcal{J} << \widetilde{W}$.
- Since T_c is a smooth function of $\delta t/\mathcal{J}$, it is reasonable to extrapolate for $\delta t \sim \mathcal{J}$.
- $\Rightarrow$ $T_c^{\max} \propto \Delta_s$ $\Rightarrow$high T_c.
- This is in contrast to the exponentially small T_c as obtained in a BCS-like mechanism.

Stripe Phases and the Mechanism of HTSC



- The broken line is the spin gap $\Delta_s(x)$ as a function of doping x
- $x_c(2)$ and $x_c(4)$: SC-CDW QPT for period 2 and period 4
- For $x \gtrsim x_c$ the isolated ladders do not have a spin gap

The Pair Density Wave and Dynamical Layer Decoupling

Li *et al* (2007): Dynamical Layer Decoupling in LBCO

- ARPES: anti-nodal d-wave SC gap is large and unsuppressed at 1/8
- Static charge stripe order for $T < T_{charge} = 54$ K
- Static Stripe Spin order $T < T_{spin} = 42$K
- ρ_{ab} drops rapidly to zero from T_{spin} to T_{KT}
- ρ_{ab} shows KT behavior for $T_{spin} > T > T_{KT}$
- $\rho_c \uparrow$ as $T \downarrow$ for $T > T^{**} \approx 35$K
- $\rho_c \to 0$ as $T \to T_{3D} = 10$K
- $\rho_c / \rho_{ab} \to \infty$ for $T_{KT} > T > T_{3D}$
- Meissner state only below $T_c = 4$K

The Pair Density Wave and Dynamical Layer Decoupling

How Do We Understand This Remarkable Effects?

- Broad temperature range, $T_{3D} < T < T_{2D}$ with 2D superconductivity but not in 3D, as if there is not interlayer Josephson coupling
- In this regime there is both striped charge and spin order
- This can only happen if there is a special symmetry of the superconductor in the striped state that leads to an almost complete cancellation of the c-axis Josephson coupling.

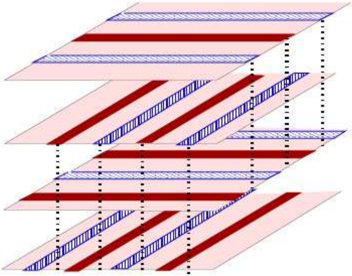
The Pair Density Wave and Dynamical Layer Decoupling

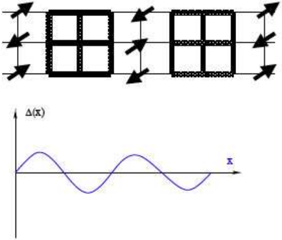
A Striped Textured Superconducting Phase

- The stripe state in the LTT crystal structure has two planes in the unit cell.
- Stripes in the 2nd neighbor planes are shifted by half a period to minimize the Coulomb interaction: 4 planes per unit cell
- The AFM spin order suffers a π phase shift accross the charge stripe which has period 4
- We propose that the superconducting order is also striped and also suffers a π phase shift.
- The superconductivity resides in the spin gap regions and there is a π phase shift in the SC order across the AFM regions

The Pair Density Wave and Dynamical Layer Decoupling

Period 4 Striped Superconducting State





- This state has intertwined striped charge, spin and superconducting orders.
- A state of this type was found in variational Monte Carlo (Ogata *et al* 2004) and MFT (Poilblanc *et al* 2007)

The Pair Density Wave and Dynamical Layer Decoupling

How does this state solve the puzzle?

- If this order is perfect, the Josephson coupling between neighboring planes cancels exactly due to the symmetry of the periodic array of π textures
- The Josephson couplings J_1 and J_2 between planes two and three layers apart also cancel by symmetry. ($J_1 / J_0 \sim 10^{-5}$ in $La_2 Cu O_4$)
- The first non-vanishing coupling J_3 occurs at four spacings. It is quite small and it is responsible for the non-zero but very low T_c
- Defects and/or discommensurations gives rise to small Josephson coupling J_0 neighboring planes

Are there other interactions?

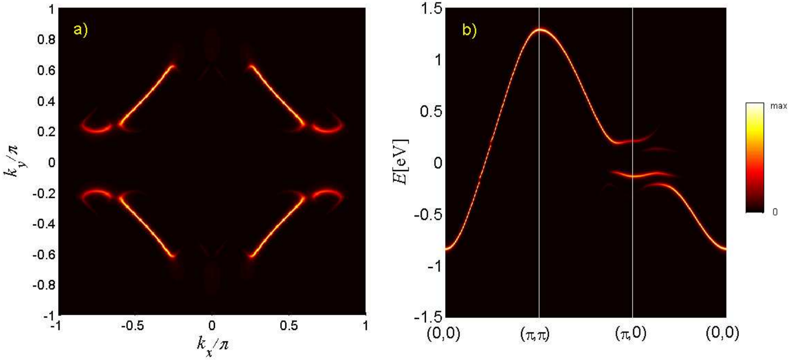
- It is possible to have an inter-plane biquadratic coupling involving the product SC of the order parameters between neighboring planes $\Delta_1 \Delta_2$ and the product of spin stripe order parameters also on neighboring planes $\mathbf{M}_1 . \mathbf{M}_2$
- However in the LTT structure $\mathbf{M}_1 . \mathbf{M}_2=0$ and there is no such coupling
- In a large enough perpendicular magnetic field it is possible (spin flop transition) to induce such a term and hence an effective Josephson coupling.
- Thus in this state there should be a strong suppression of the 3D SC T_c but not of the 2D SC T_c

The Pair Density Wave and Dynamical Layer Decoupling

Away from x=1/8

- Away from x=1/8 there is no perfect commensuration
- Discommensurations are defects that induce a finite Josephson coupling between neighboring planes $J_1 \sim |x-1/8|^2$, leading to an increase of the 3D SC T_c away from 1/8
- Similar effects arise from disorder which also lead to a rise in the 3D SC T_c

Quasiparticle Spectral Function of the Striped Superconductor



The Pair Density Wave and Dynamical Layer Decoupling

Landau-Ginzburg Theory of the striped SC: Order Parameters

- Striped SC: $\Delta(\mathbf{r})=\Delta_{\mathbf{Q}}(\mathbf{r})\, e^{i\,\mathbf{Q}\cdot\mathbf{r}}+\Delta_{-\mathbf{Q}}(\mathbf{r})\, e^{-i\mathbf{Q}\cdot\mathbf{r}}$, a singlet pair condensate with wave vector, (*i.e.* an FFLO type state at zero magnetic field), a complex charge 2 scalar field.
- Nematic: detects breaking of rotational symmetry: N, a real neutral pseudo-scalar field
- Charge stripe: $\rho_{\mathbf{K}}$, unidirectional charge stripe with wave vector $\mathbf{K}$
- Spin stripe order parameter: $\mathbf{S}_{\mathbf{Q}}$, a neutral complex spin vector field, $\mathbf{K}=2\mathbf{Q}$

The Pair Density Wave and Dynamical Layer Decoupling

Ginzburg-Landau Free Energy Functional

- $F = F_2 + F_3 + F_4 + \ldots$
- The quadratic and quartic terms are standard
- $F_3 = \gamma_s\, \rho_{\mathbf{K}}^*\, \mathbf{S}_{\mathbf{Q}} \cdot \mathbf{S}_{\mathbf{Q}} + \pi/2 \text{ rotation} + \text{c.c.}$
  $+\gamma_\Delta\, \rho_{\mathbf{K}}^*\, \Delta_{-\mathbf{Q}}^*\, \Delta_{\mathbf{Q}} + \pi/2 \text{ rotation} + \text{c.c.}$
  $+g_\Delta\, N\, (\Delta_{\mathbf{Q}}^*\, \Delta_{\mathbf{Q}} + \Delta_{-\mathbf{Q}}^*\, \Delta_{-\mathbf{Q}} - \pi/2 \text{ rotation}) + \text{c.c.}$
  $+g_s\, N\, (\mathbf{S}_{\mathbf{Q}}^* \cdot \mathbf{S}_{\mathbf{Q}} - \pi/2 \text{ rotation})$
  $+g_c\, N\, (\rho_{\mathbf{K}}^*\, \rho_{\mathbf{K}} - \pi/2 \text{ rotation})$

The Pair Density Wave and Dynamical Layer Decoupling

Some Consequences of the GL theory

- The symmetry of the term coupling charge and spin order parameters requires the condition K= 2Q
- Striped SC order implies charge stripe order with 1/2 the period, and of nematic order
- Charge stripe order with wave vector 2Q and/or nematic order favors stripe superconducting order which may or may not occur depending on the coefficients in the quadratic part

Effects of Disorder

- The striped SC order is very sensitive to disorder: disorder ⇒ pinned charge density wave ⇒ coupling to the phase of the striped SC ⇒ SC "gauge" glass with zero resistance but no Meissner effect in 3D
- Disorder induces dislocation defects in the stripe order
- Due to the coupling between stripe order and SC, $\pm\pi$ flux vortices are induced at the dislocation core.
- Strict layer decoupling only allows for a magnetic coupling between randomly distributed $\pm\pi$ flux vortices
- Novel glassy physics and "fractional" flux

Charge 4e SC order

- Coupling to a charge 4e SC order parameter Δ_4
- $F'_3 = g_4\,[\Delta_4^* \,(\Delta_Q \Delta_{-Q} + \text{rotation}) + \text{c.c.}]$
- Striped SC order (PDW) ⇒ uniform charge 4e SC order!
- Since the coupling is independent of θ_-, the charge 4e SC order is unaffected by the Bragg glass of the pinned CDW
- The half vortices of θ_+ are the fundamental hc/4e vortices of the charge 4e SC.

The Pair Density Wave and Dynamical Layer Decoupling

Coexisting uniform and striped SC order

- PDW order Δ_Q and uniform SC order Δ_0
- $F_{3,u} = \Upsilon_\Delta \Delta_0^* \rho_Q \Delta_{-Q} + \rho_{-Q} \Delta_Q + g_\rho \rho_{-2Q} \rho_Q^2 + \text{rotation} + \text{c.c.}$
- If $\Delta_0 \neq 0$ and $\Delta_Q \neq 0 \Rightarrow$ there is a ρ_Q component of the charge order!
- The small uniform component Δ_0 removes the sensitivity to quenched disorder of the PDW state

The Pair Density Wave and Dynamical Layer Decoupling

Topological Excitations of the Striped SC

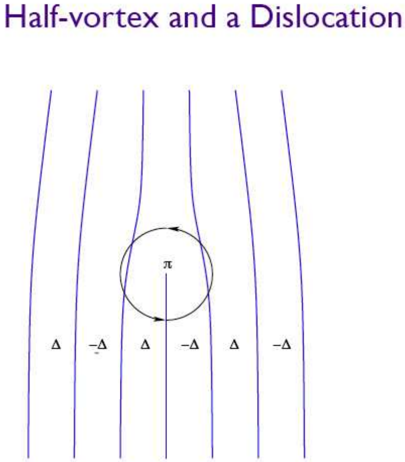
- $\rho(r)=|\rho_K| \cos [K\, r+ \Phi(r)]$
- $\Delta(r)=|\Delta_Q| \exp[i\, Q\, r + i\, \theta_Q(r)]+|\Delta_{-Q}| \exp[-i\, Q\, r + i\, \theta_{-Q}(r)]$
- $F_{3,\Upsilon}=2\Upsilon_\Delta\, |\rho_K\, \Delta_Q\, \Delta_{-Q}| \cos[2\, \theta_-(r)-\Phi(r)]$
- $\theta_{\pm Q}(r)=[\theta_+(r) \pm \theta_-(r)]/2$
- $\theta_{\pm Q}$ single valued mod $2\pi \Rightarrow \theta_\pm$ defined mod π
- ϕ and θ_- are locked $\Rightarrow$ topological defects of ϕ and θ_+

Topological Excitations of the Striped SC

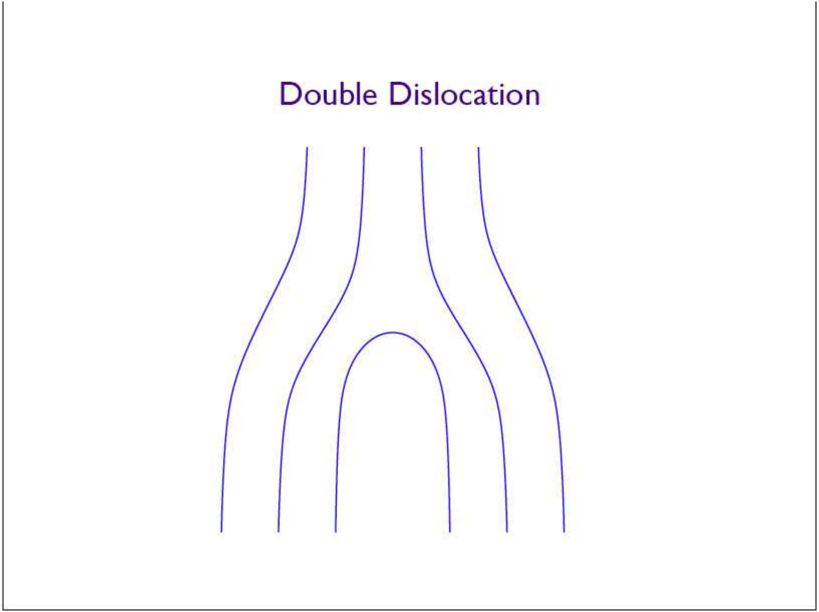
- SC vortex with $\Delta\theta_+ = 2\pi$ and $\Delta\phi=0$
- Bound state of a 1/2 vortex and a dislocation

  $\Delta\theta_+ = \pi, \Delta\phi = 2\pi$
- Double dislocation, $\Delta\theta_+ = 0, \Delta\phi = 4\pi$
- All three topological defects have logarithmic interactions

The Pair Density Wave and Dynamical Layer Decoupling



The Pair Density Wave and Dynamical Layer Decoupling

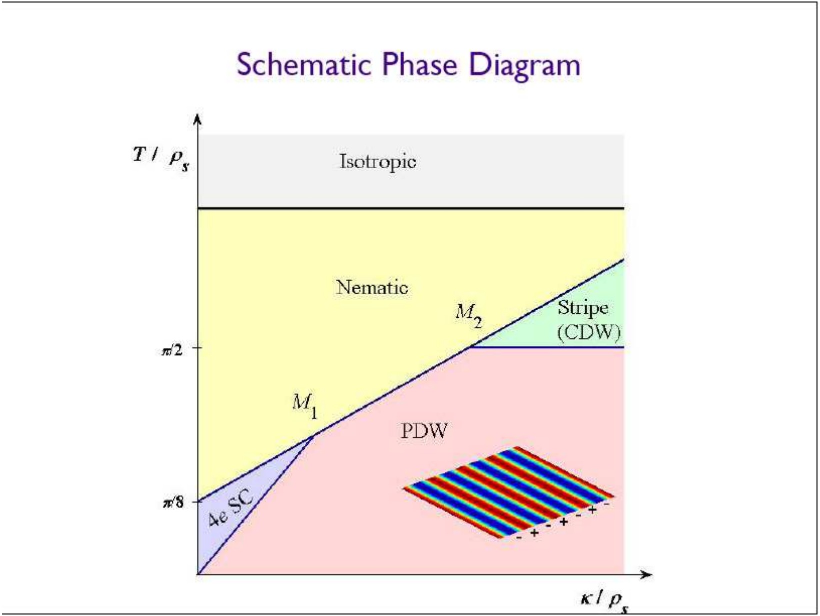


The Pair Density Wave and Dynamical Layer Decoupling

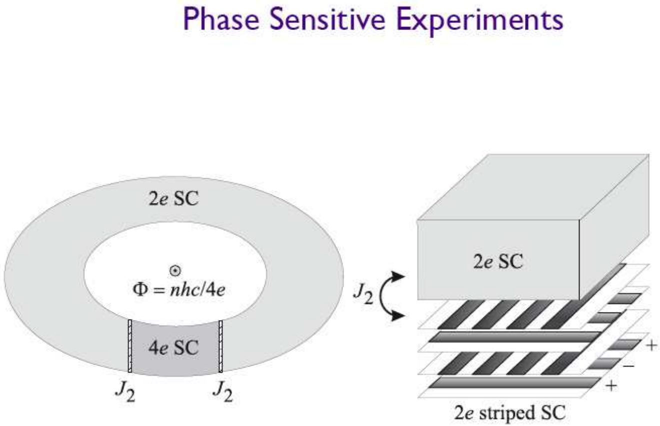
Thermal melting of the PDW state

- Three paths for thermal melting of the PDW state
- Three types of topological excitations: (1,0) (SC vortex), (0,1) (double dislocation), ($\pm 1/2, \pm 1/2$) (1/2 vortex, single dislocation bound pair)
- Scaling dimensions: $\Delta_{p,q}=\pi(\rho_{sc}\, p^2+\kappa_{cdw}\, q^2)/T=2$ (for marginality)
- Phases: PDW, Charge 4e SC, CDW, and normal (Ising nematic)

The Pair Density Wave and Dynamical Layer Decoupling



The Pair Density Wave and Dynamical Layer Decoupling

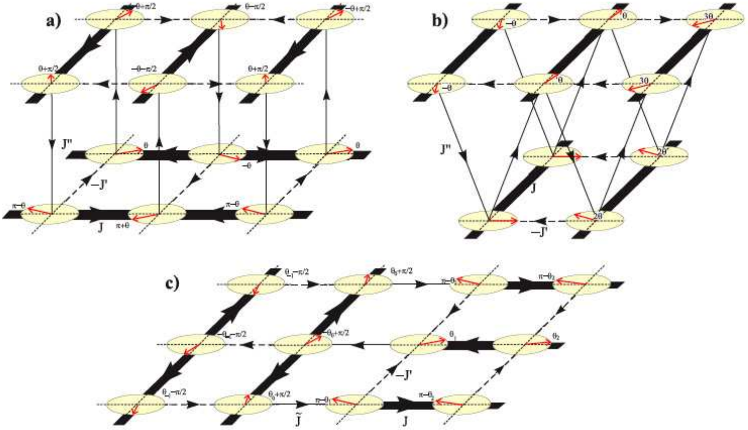


The Pair Density Wave and Dynamical Layer Decoupling

Non-collinear Order and Time Reversal Symmetry Breaking

- PDW order in the planes leads to frustration of the inter-plane Josephson coupling
- We will regard the SC phase as an XY "pseudo-spin" and we get non-collinear order
- Non-collinear order $\Leftrightarrow$ circulating currents $\Rightarrow$ Time Reversal symmetry breaking effects
- The resulting non-collinear order depends on the lattice, and it is different for the LTT structure of LBCO than for the orthorhombic (chain) structure of YBCO
- Twin boundaries (domain walls) and edges also lead to non-collinear order
- The period of the Josephson coupling between a uniform and a striped SC is π

The Pair Density Wave and Dynamical Layer Decoupling



Non-collinear order and circulating currents for (a) the LTT structure (LBCO), (b) the orthorhombic (chain) structure of YBCO, and (c) an in-plane domain wall ("twin boundary").