

Entanglement and quantum noise: Is it possible to measure entanglement entropies?

Colloquium at the Perimeter Institute, Waterloo, Ontario (Canada),
April 27, 2011

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April 25, 2011

Collaborators and References

- ▶ Benjamin Hsu
- ▶ Eytan Grosfeld
- ▶ Benjamin Hsu, Eytan Grosfeld, and Eduardo Fradkin, *Quantum noise and entanglement generated by a local quantum quench*, Phys. Rev. B **80**, 235412 (2009), arXiv:0908.2622.
- ▶ Eduardo Fradkin, *Scaling of Entanglement Entropy at 2D quantum Lifshitz fixed points and topological fluids*, Journal of Physics A: Mathematical and Theoretical **42**, 504011 (2009), (special issue on Entanglement Entropy, P. Calabrese, J. Cardy and B. Doyon, editors); arXiv:0906.1569v1.

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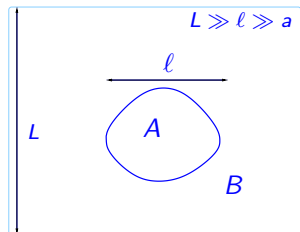
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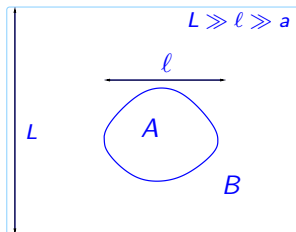
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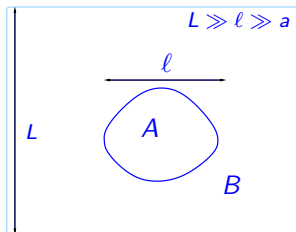
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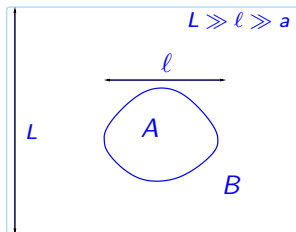
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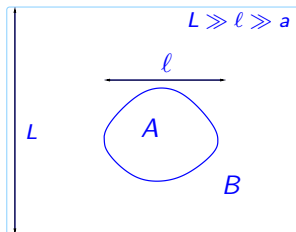


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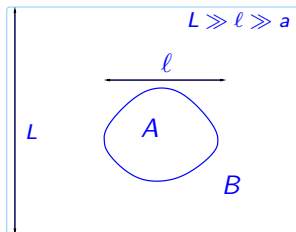
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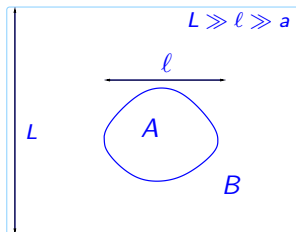
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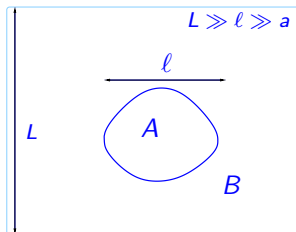
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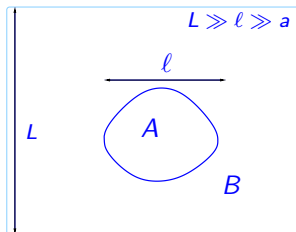
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The excitations are vortices with fractional charge $q = e/m$ and fractional (braid) statistics $\theta = \pi/m$.

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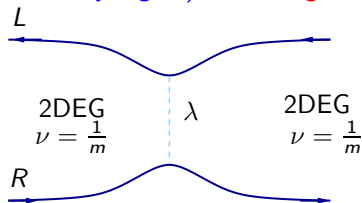
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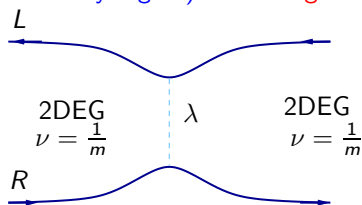
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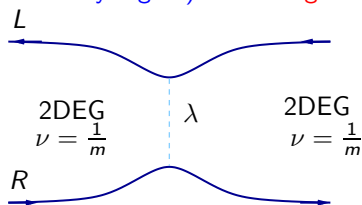
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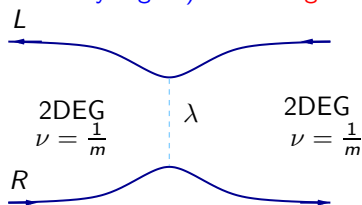
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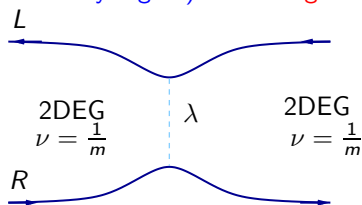
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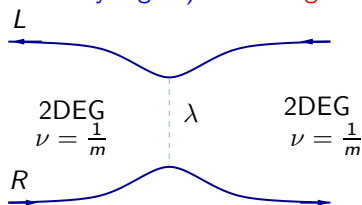
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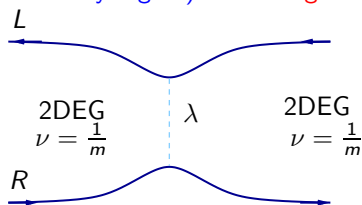
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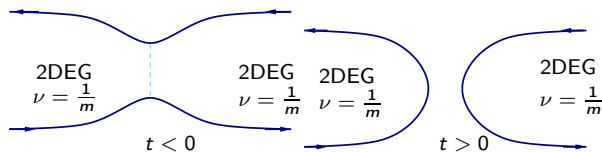
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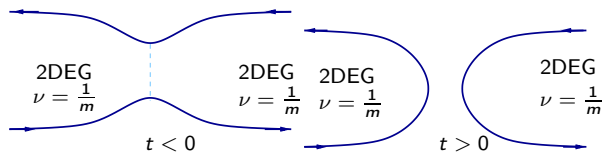
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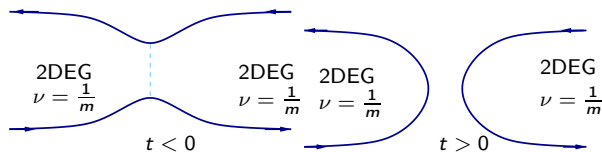
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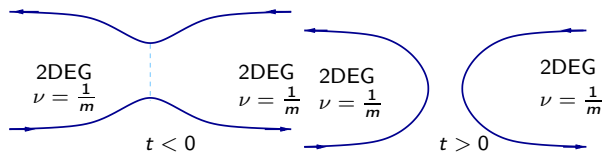
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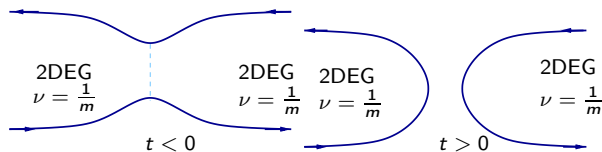
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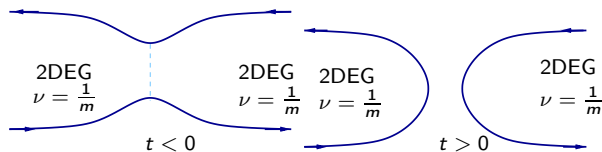
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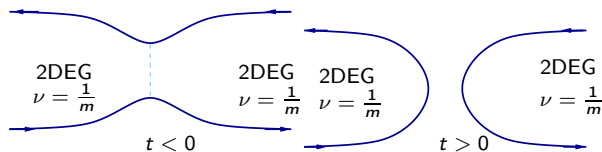
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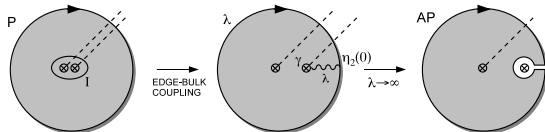
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Two σ_2 operators are drawn from the vacuum. Tunneling (of strength λ) is introduced between one of the σ_2 operators and the edge. Finally, in the limit $\lambda \rightarrow \infty$, the edge circumvents the σ_2 operator.

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Local quench in the Ising spin chain

$$H = \sum_{n=-N/2}^{N/2+1} \sigma_1(n) + \lambda \sum_{n=-N/2}^{N/2+1} \sigma_3(n)\sigma_3(n+1)$$

- ▶ @ link (0, 1): $\lambda(t) = 0$ for $t < 0$ and $\lambda(t) = 1$ for $t > 0$
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- ▶ Majorana fermion description of the critical quantum Ising model

$$\mathcal{L} = i\tilde{\eta}_R(\partial_t - \partial_x)\tilde{\eta}_R + i\eta_L(\partial_t - \partial_x)\eta_L + i\lambda(t)\delta(x)\eta_L\eta_R, \quad \tilde{\eta}_R(x) = \eta_R(-x)$$

$$\text{B.C: } \tilde{\eta}_R(0^+) = -\tilde{\eta}_L(0^-)$$

- ▶ Locally conserved energy density: $\rho_E(x) = \eta_L i\partial_x \eta_L - \tilde{\eta}_R i\partial_x \tilde{\eta}_R$
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