Can you hear the shape of Schrödinger's cat?

Colloquium at the Department of Physics of the University of Illinois,December 4, 2008

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Department of Physics
University of Illinois at Urbana Champaign

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Outline

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- Topological phases in condensed matter and quantum phase transitions

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- Topological quantum computing?
- Conclusions

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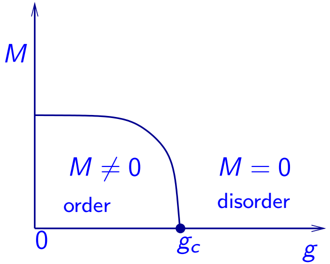

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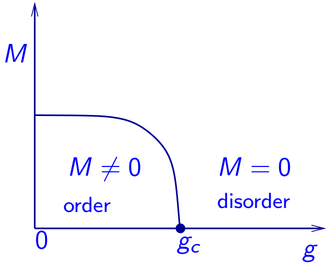

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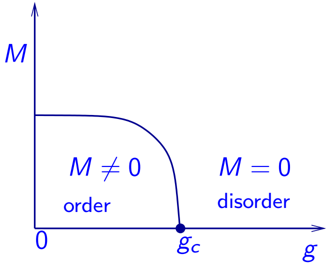

- Order parameter: $M = \langle \sigma_z \rangle$ $\Rightarrow$ *spontaneous symmetry breaking*
- Quantum Critical Point g_c: *scale invariance*, correlation length: $\xi \sim |g - g_c|^{-\nu}$, energy gap: $m \sim |g - g_c|^{z\nu}$ $z = 1$ for the Ising model

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- Natural candidate: Quantum Entanglement, measured by the von Neumann Entanglement Entropy (to be defined shortly...)

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- Effective field theory description: Topological Field Theory, *e.g.*, Chern-Simons gauge theory, discrete gauge theory.

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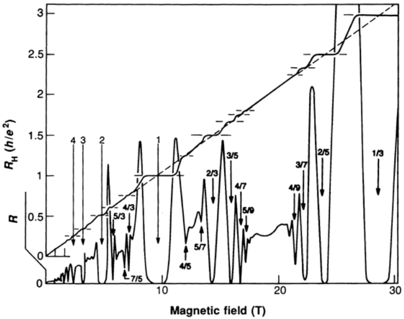

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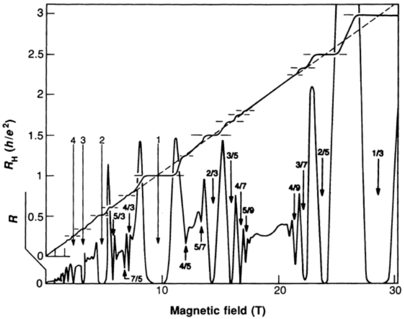

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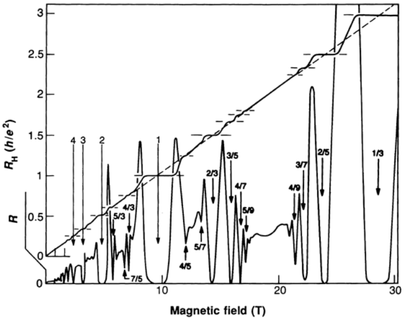

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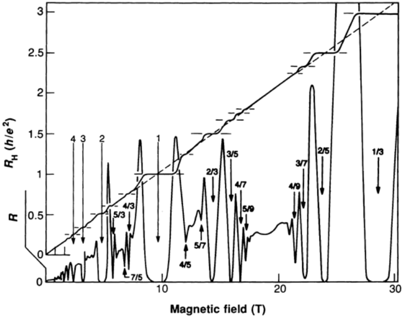

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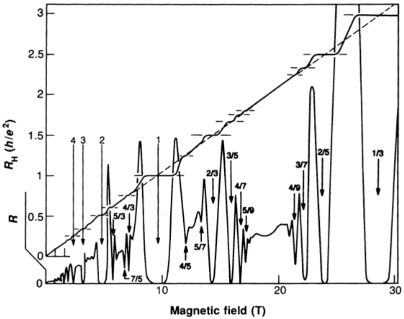

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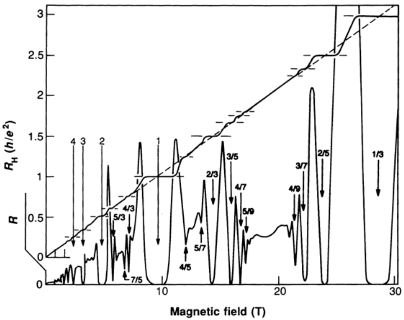

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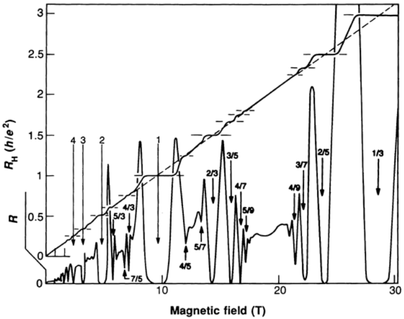

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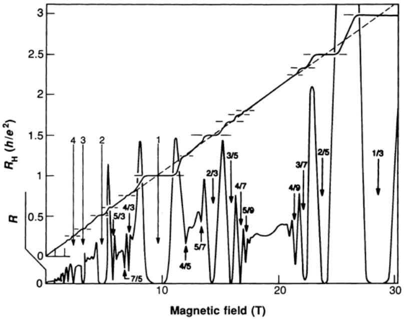

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- Time-Reversal Breaking Superconductors: Sr_2RuO_4 is a $p_x + ip_y$ superconductor (strong evidence, not uncontroversial)

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- The excitations are vortices with fractional charge $q = e/m$ and fractional (braid) statistics $\theta = \pi/m$.

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    - Laughlin vortices with charge e/m and abelian fractional statistics π/m

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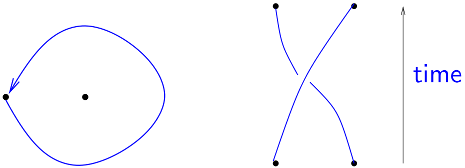

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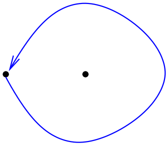

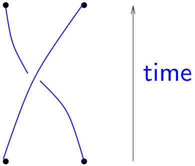

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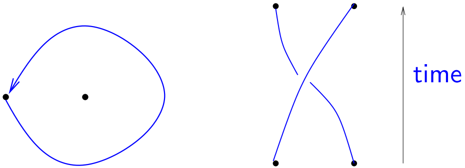

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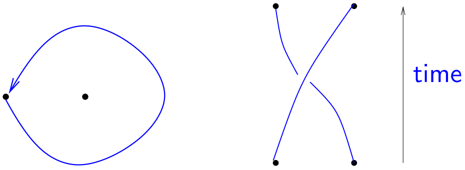

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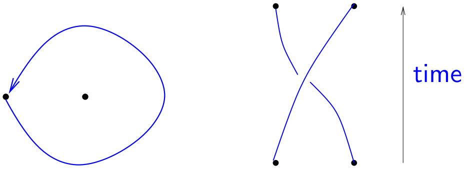

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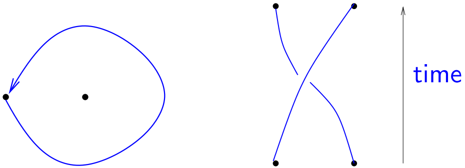

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Anyons with Abelian (braid) fractional statistics!

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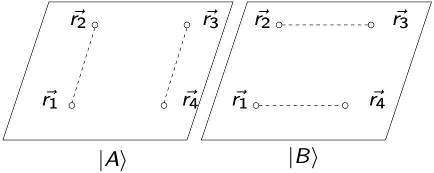

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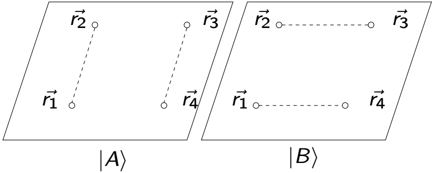

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- Four non-Abelian anyons can be regarded as a topological qubit

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  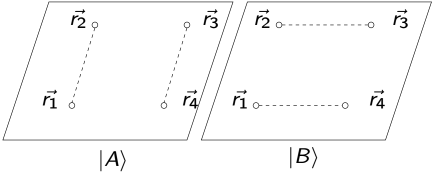

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  $|A\rangle \to a|A\rangle + b|B\rangle$
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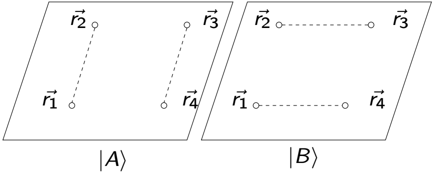

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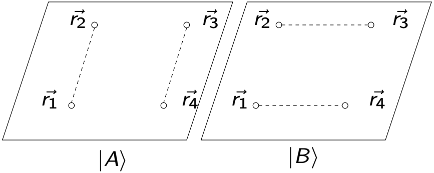

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$$\text{Braiding matrix} \Rightarrow \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

FQH Interferometers and Fractional Statistics

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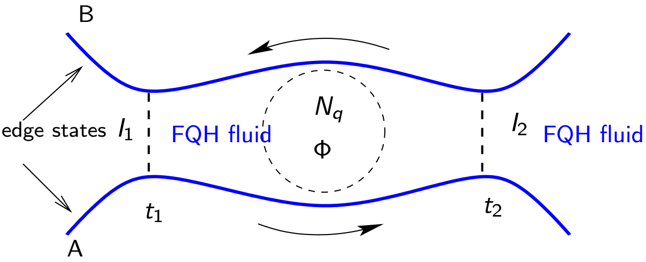

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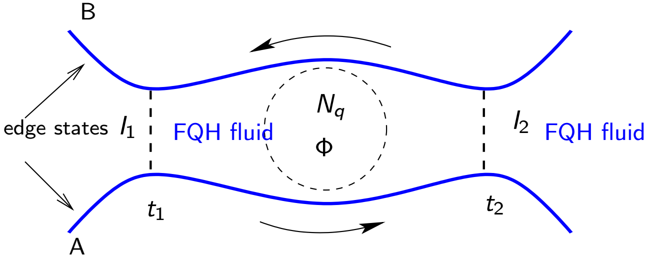

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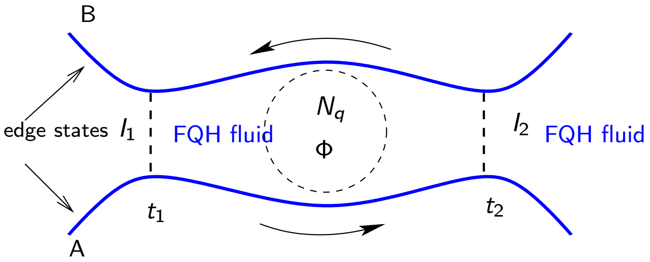

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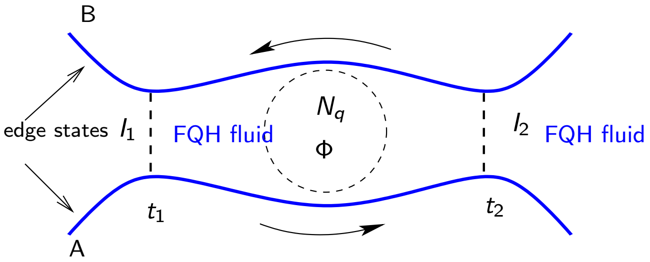

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- If, on the other hand, we vary N_q, we can probe the statistics.

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- Basis of current proposals (Das Sarma, Freedman and Nayak) to construct a "topological qubit"

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- square lattice: quantum critical point; triangular lattice: Topological $\mathbb{Z}_2$ deconfined phase (Moessner and Sondhi, 1998)

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- Matching the correlation functions of the Quantum Dimer and Lifshitz models, one finds $\kappa = \frac{1}{4\pi}$.

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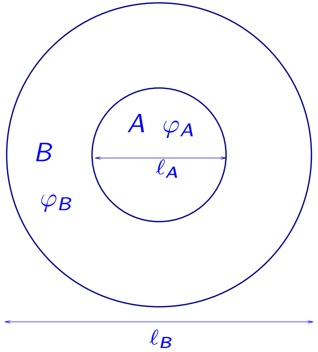

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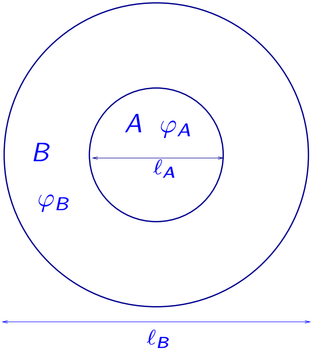

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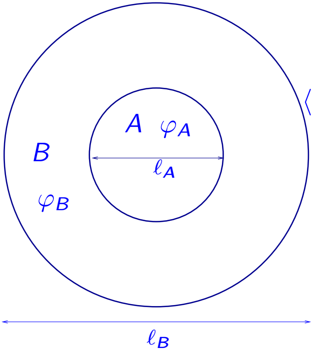

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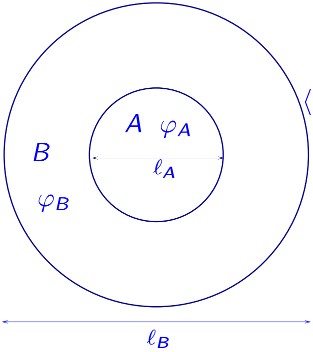

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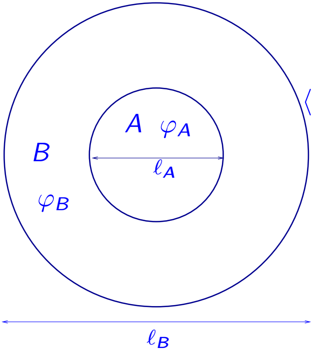

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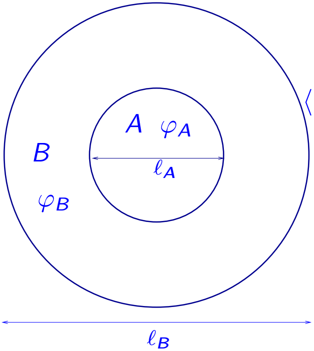

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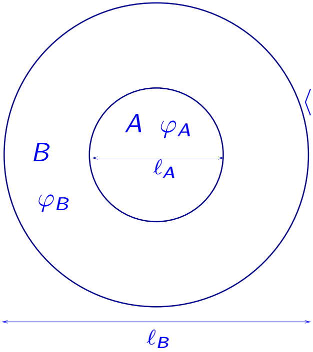

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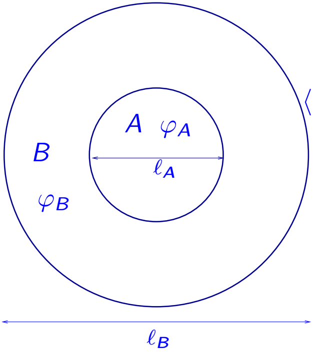

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Entanglement Entropy of Scale Invariant Wave Functions: Can you hear the shape of Schrödinger's cat?

with Joel Moore

Physics Illinois

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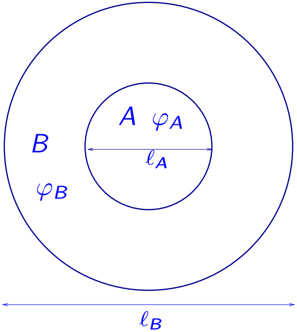

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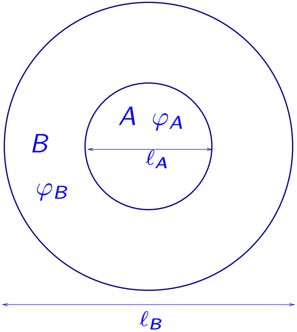

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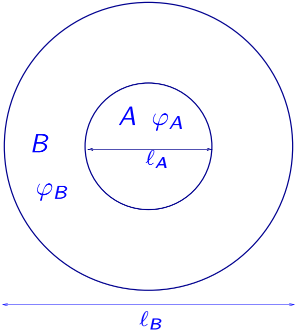

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- If the $\ln \ell$ term cancels $\Rightarrow$ the O(1) term is universal and determined by the CFT

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  (There are also geometry-dependent scale invariant terms)
  Hsu, Mulligan, Fradkin and Kim (2008)

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