Problem 1  The Dirac Equation

1. Before this we recall the Dirac equations for $\psi$ and $\bar{\psi}$

$$-m\bar{\psi} - i\bar{\psi}\partial = 0$$
$$m\psi - i\partial\psi = 0$$

We can take the partial derivative of the 4-current with respect to the general 4-vector, $X_\mu$.

$$\partial_\mu j^\mu = \partial_\mu (\bar{\psi}\gamma^\mu\psi)$$
$$= \partial_\mu \bar{\psi}\gamma^\mu\psi + \bar{\psi}\partial^\mu(\gamma^\mu\psi)$$

From Dirac equation

$$\partial_\mu j^\mu = im\bar{\psi}\psi + (-im\bar{\psi}\psi)$$
$$= 0$$

2. From Dirac equation

$$(i\partial - m)\psi = 0$$
$$(i\partial + m)(i\partial - m)\psi = 0$$
$$(-\partial\partial - m^2)\psi = 0$$

$$\partial\partial = \partial_\mu\partial_\nu \frac{1}{2}([\gamma^\mu, \gamma^\nu] + \{\gamma^\mu, \gamma^\nu\})$$
$$= \partial_\mu\partial_\nu g^{\mu\nu}$$

It is simple to see that the first term is equal to zero by swapping the indices $\partial_\mu\partial_\nu[\gamma^\mu, \gamma^\nu] = -\partial_\nu\partial_\mu[\gamma^\mu, \gamma^\nu]$ Therefore we are left with

$$(-\partial_\mu\partial^\mu - m^2)\psi = 0$$
which is the Klein-Gordon equation.

3. (a) From the Clifford algebra

\[
\gamma^\mu \gamma^\nu = \frac{1}{2} [\gamma^\mu, \gamma^\nu] + \frac{1}{2} \{ \gamma^\mu, \gamma^\nu \}
\]

\[
= g^{\mu\nu} - i\sigma^{\mu\nu}
\]

Now

\[
A^\mu B^\nu \gamma^\mu \gamma^\nu = A^\mu B^\nu g_{\mu\nu} - i\sigma_{\mu\nu} A^\mu B^\nu = A \cdot B - i\sigma_{\mu\nu} A^\mu B^\nu
\]

(b) First note that this trace operator is over the spinor indices and The A and B have no spinor indices

\[
tr \mathcal{AB} = tr(\gamma^\mu \gamma^\nu A_\mu B_\nu)
\]

\[
= A_\mu B_\nu \frac{1}{2} tr(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu) + A_\mu B_\nu \frac{1}{2} tr\sigma^{\mu\nu}
\]

The second contribution vanishes since \(\sigma^{\mu\nu}\) is traceless. Hence,

\[
tr \mathcal{AB} = A^\mu B^\nu tr g_{\mu\nu}
\]

\[
= A \cdot B tr I = 4 A \cdot B.
\]

(c)

\[
\gamma^\lambda \gamma^\mu \gamma^\lambda = (-\gamma^\mu \gamma^\lambda + 2g^{\mu\lambda})\gamma^\lambda
\]

\[
= -\gamma^\mu (g^{\lambda\mu} g_{\mu\lambda} - i\sigma^{\lambda\mu} g_{\mu\lambda}) + 2\gamma^\mu
\]

\[
= -4\gamma^\mu + 2\gamma^\mu
\]

\[
= -2\gamma^\mu
\]

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Problem 2 Transformation Properties of Field Bilinears in the Dirac Theory

Some properties we learned in class are important. First we found that Lorentz transformation induces a similarity transformation on the \(\gamma\) matrices that is equivalent to multiplying it by an inverse Lorentz transformation.

\[
S(\Lambda)\gamma^\mu S^{-1}(\Lambda) = (\Lambda^{-1})_\mu^\nu \gamma^\nu
\]

Also that the \(\gamma\) matrices are frame independent. First we want to know how \(\bar{\psi}'(x')\) transforms.

\[
\bar{\psi}'(x') = \psi'^\dagger(x')\gamma^0 = \psi^\dagger(x)(1 - \frac{i}{4}\omega^{\mu\nu}\sigma^\dagger_{\mu\nu} + ...)\gamma^0
\]

We have to see whether \(\gamma^0\) commutes with the \(\sigma^\dagger_{\mu\nu} = \frac{i}{4}[\gamma^\dagger_{\mu}, \gamma^\dagger_{\nu}]\). There are two cases. When \(\mu, \nu \neq 0\), \(\sigma^\dagger_{\mu\nu} = \gamma^0\sigma_{\mu\nu}\gamma^0\). If one of the indices is zero then \(\sigma^\dagger_{\mu\nu} = -\sigma_{\mu\nu}\) and \(\gamma^0\) anticommutes. Essentially we have

\[
\bar{\psi}'(x') = \psi^\dagger\gamma^0(1 - \frac{i}{4}\omega^{\mu\nu}\sigma_{\mu\nu} + ...) = \bar{\psi}(x)S^{-1}(\Lambda)
\]
1. \( \bar{\psi}(x') \psi'(x') = \bar{\psi}(x) S^{-1}(\Lambda) S(\Lambda) \psi(x) = \bar{\psi}(x) \psi(x) \)

2. \[
\bar{\psi}(x') \gamma^5 \psi'(x') = \bar{\psi}(x) S^{-1}(\Lambda) \gamma^5 S(\Lambda) \psi(x) \\
= \bar{\psi}(x) S^{-1}(\Lambda) \frac{i}{4!} \epsilon_{\mu\nu\lambda\sigma} \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma S(\Lambda) \psi(x) \\
= \bar{\psi}(x) \frac{i}{4!} \epsilon_{\mu\nu\lambda\sigma} \Lambda^\nu_\alpha \Lambda^\lambda_\beta \Lambda^\sigma_\rho \gamma^\alpha \gamma^\beta \gamma^\epsilon \psi(x) \\
= \bar{\psi}(x) i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \frac{i}{4!} \epsilon_{\mu\nu\lambda\sigma} \Lambda^\nu_0 \Lambda^\lambda_1 \Lambda^\sigma_2 \psi(x) \\
= \bar{\psi}(x) \gamma_5 \text{det}(\Lambda) \psi(x)
\]

3. \( \bar{\psi}'(x') \gamma^\mu \psi'(x') = \bar{\psi}'(x) S^{-1} \gamma^\mu S \psi(x) = \Lambda^\nu_\mu \bar{\psi}(x) \gamma^\nu \psi(x) \)

4. \( \bar{\psi}'(x') \gamma_5 \gamma^\mu \psi'(x') = \bar{\psi}(x) S^{-1} \gamma_5 S S^{-1} \gamma^\mu S \psi(x) = \text{det}(\Lambda) \Lambda^\nu_\mu \bar{\psi}(x) \gamma_5 \gamma^\nu \psi(x) \)

5. \( \bar{\psi}'(x') \sigma^{\mu\nu} \psi'(x') = \bar{\psi}(x) S^{-1} \frac{i}{2} [\gamma^\mu, \gamma^\nu] S \psi(x) \\
= \Lambda^\mu_\alpha \Lambda^\nu_\beta \bar{\psi}(x) \sigma^{\alpha\beta} \psi(x) \)

The second line comes from inserting \( SS^{-1} \) in between each \( \gamma \) matrices.

\[ \cdots \cdots \]

**Problem 3  Chiral Symmetry**

1. Dirac equation in chiral basis becomes

\[
i \partial_0 \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} - i \partial_i \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} - m \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} = 0 \\
\begin{pmatrix} -m & -i \partial_0 I - i \partial_i \sigma^i \\ -i \partial_0 I + i \partial_i \sigma^i & -m \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix} = 0
\]

From here we get two coupled equations of the 2-spinors.

2. If the \( m=0 \) then the equation decouples into two 2x2 equation namely

\[
i \begin{pmatrix} \partial_0 + \partial_3 & \partial_1 + i \partial_2 \\ \partial_1 - i \partial_2 & \partial_0 - \partial_3 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = 0 \\
i \begin{pmatrix} \partial_0 - \partial_3 & -\partial_1 + i \partial_2 \\ -\partial_1 - i \partial_2 & \partial_0 + \partial_3 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = 0
\]
Plugging in plane wave ansatz: \( \chi_i \sim e^{ik_i x^\mu} \) and \( \phi_i \sim e^{ip_i x^\mu} \), one finds

\[
i \begin{pmatrix} k_0 + kl_3 & \partial_1 - i\partial_2 \\ k_1 - ik_2 & k_0 - k_3 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = 0
\]

\[
i \begin{pmatrix} p_0 - p_3 & -p_1 + ip_2 \\ -p_1 - ip_2 & p_0 + p_3 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = 0
\]

These equations have non-trivial answers when the determinant equal to zero, i.e. \( k_0^2 = |\vec{k}|^2; p_0^2 = |\vec{p}|^2 \). Recall that the 0-component of the momentum four-vector is the energy. Hence, we find \( E = \pm |\vec{p}| \), which is the dispersion relation for a massless relativistic particle.

One can solve for each component for \( \chi_1, \chi_2 \) and \( \phi_1, \phi_2 \). One finds

\[
\chi_1 = -\frac{k_1 - ik_2}{k_0 + k_3} \chi_2
\]

or

\[
\chi_2 = -\frac{k_1 + ik_2}{k_0 - k_3} \chi_1
\]

Similar solutions can be found for \( \phi \). To assign chirality, note that \( \gamma_5 \) has eigenvalues \( \pm 1 \). Note that the operators, \( P_+ = \frac{I + \gamma_5}{2} \); \( P_- = \frac{I - \gamma_5}{2} \). If we act these operators on \( \psi \), we find that \( \phi \) lives in a vector space with eigenvalue +1 and \( \chi \) lives in the vector space with eigenvalue -1.

3. Consider the transformation in the matrix form

\[
CT = e^{i\gamma_5 \theta} = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}
\]

You can show this easily by Taylor expansion and the fact that \( (\gamma_5)^2 = I \). It is also easy in matrix form to verify that

\[
(e^{i\gamma_5 \theta})^\dagger \gamma_0 = \gamma_0 e^{i\gamma_5 \theta}
\]

(a)

\[
CT \times \psi = \begin{pmatrix} e^{i\theta} \phi \\ e^{-i\theta} \chi \end{pmatrix}
\]

(b) Immediately from eq.1, \( \bar{\psi} \rightarrow \bar{\psi} e^{i\gamma_5 \theta} \)

(c) From (a) and (b) it follows that \( \bar{\psi} \gamma^\mu \psi \rightarrow \bar{\psi} e^{2i\gamma_5 \theta} \psi \)

To find how \( \bar{\psi} \gamma^\mu \psi \) transforms, it is easier to look at the Taylor expansion and use the fact that \( \{\gamma_5, \gamma^\mu\} = 0 \). Hence,

\[
\gamma^\mu e^{i\gamma_5 \theta} = \gamma^\mu (I + i\gamma_5 \theta - I \frac{\theta^2}{2} - i\gamma_5 \frac{\theta^3}{3!} + \ldots) = (I - i\gamma_5 \theta - I \frac{\theta^2}{2} + i\gamma_5 \frac{\theta^3}{3!} + \ldots) \gamma^\mu = e^{-i\gamma_5 \theta} \gamma^\mu
\]

Hence,

\[
\bar{\psi} \gamma^\mu \psi \rightarrow \bar{\psi} e^{i\gamma_5 \theta} \gamma^\mu e^{i\gamma_5 \theta} \psi \rightarrow \bar{\psi} e^{i\gamma_5 \theta} e^{-i\gamma_5 \theta} \gamma^\mu \psi \rightarrow \bar{\psi} \gamma^\mu \psi
\]

(d) From the transformation property of \( \psi \psi \), we can conclude that the Dirac equation is not covariant under a CT if \( m \neq 0 \). The new terms find are the mass term with the phase factor \( e^{i2\gamma_5 \theta} \).
Problem 4 The Landau Theory of Phase Transitions as a Classical Field Theory

1. By Varying the free energy for the small change in the order parameter field

\[
\delta F = \int d^d x \delta E = \int d^d x \left( \frac{\delta E}{\delta \phi} \delta (\partial_i \phi) + \frac{\delta E}{\delta \phi} \delta \phi \right) = \int d^d x \left( \frac{\delta E}{\delta \partial_i \phi} \partial_i (\delta \phi) + \frac{\delta E}{\delta \phi} \delta \phi \right)
\]

The variation vanishes at the boundary. The saddle-point equation is

\[
-\partial_i \frac{\delta E}{\delta \partial_i \phi} + \frac{\delta E}{\delta \phi} = 0 \rightarrow \nabla^2 \phi - \frac{\delta U(\phi)}{\delta \phi} = 0
\]

For our specific Lagrangian, this is

\[
\nabla^2 \phi - \left( m^2 \phi + \frac{\lambda_4}{3!} \phi^3 + \frac{\lambda_6}{5!} \phi^5 \right) = 0
\] (2)

2. By letting \( \phi = \bar{\phi} \), a constant and looking at the equation of motion, we find that the gradient term is zero. Hence, the saddle point equation becomes

\[
m^2 \bar{\phi} + \frac{\lambda_4}{3!} \bar{\phi}^3 + \frac{\lambda_6}{5!} \bar{\phi}^5 = 0
\]

Values of the field which satisfy the saddle-point equations are the inflection points of the potential. One can graphically identify the points which are the true minima. We can solve the above equation for \( \phi \).

\[
\bar{\phi}^2 = -10\alpha (1 \pm \sqrt{1 - \mu}) \text{ and } \bar{\phi} = 0
\] (3)

where \( \alpha = \frac{\lambda_4}{\lambda_6} \) and \( \mu = \frac{6m^2}{5\lambda_6\alpha^2} \). For the next part, we are given that \( \lambda_4 < 0, \alpha < 0 \). Note that \( \phi \) is real so we have to explore difference cases

(a) \( \mu > 1 \) the square root is imaginary and thus not allowed. Hence, the only allowed value of the order parameter field, \( \bar{\phi} = 0 \). This corresponds to the temperature range:

\[
T > \frac{5\lambda_4^2}{6a\lambda_6} + T_0
\]

(b) \( \mu = 1 \). The square root vanishes. There are two non-zero solutions possible. This corresponds to

\[
T^* = \frac{5\lambda_4^2}{6a\lambda_6} + T_0
\] (4)

(c) \( \mu < 1 \): In this case, the square root is real, and there are 4 non-zero solutions. This case corresponds to the temperature range

\[
T < \frac{5\lambda_4^2}{6a\lambda_6} + T_0
\]
Figure 1: $U(\phi)$ for various temperatures. There are also negative solutions which are this graph reflected across the y-axis.

Figure 2: A plot of $\bar{\phi}$ as a function of temperature. Notice the sharp discontinuity in the graph.
To see whether these are maxima or a minima one can identify them from the graph. Notice the discontinuous jump when $T$ reaches $T^*$. This is a signature of the first order phase transition.

3. Consider the case $\lambda_4 > 0$. From eq.3, we see that the existence of a non-zero solution depends on the sign of $1 \pm \sqrt{1 - \mu}$. There are cases to consider:

(a) $1 \pm \sqrt{1 - \mu} > 0$ In this case only $\phi = 0$ is the solution since the other solutions are imaginary. This corresponds to the temperature:

$$ T > T_0 $$

(b) $1 \pm \sqrt{1 - \mu} = 0$, In this case only $\phi = 0$ is the solution since the other solutions are imaginary. This one is fifth root so there are degenerate solutions. This corresponds to the temperature:

$$ T = T^* = T_0 $$

(c) $1 \pm \sqrt{1 - \mu} < 0$, In this case there are non-zero solutions on top of the $\phi = 0$ solutions. This corresponds to the temperature:

$$ T < T_0 $$

In this case the order paramet decreases continuously to zero. This is a signature of the second order phase transition.

4. Inverting eq.4 yields the behavior of the phase boundary

$$ \lambda_4 \sim -(T - T_0)^{1/2} $$

In addition the phase boundary for the second order phase transition is just a vertical line up since it’s only at $T = T_0$. 

Figure 3: A plot of $U(\phi)$ as a function of $m^2$. 
Figure 4: A plot of $\bar{\phi}$ as a function of temperature. Notice the value of the order parameter goes to zero smoothly.

Figure 5: Phase Diagram. The first order phase transition’s phase boundary has a square root dependence.
Problem 5  Scalar Electrodynamics

1. Under the gauge transformations given in the problem set, we can prove the following

(a) \( |\phi(x)|^2 \rightarrow \phi^*(x)e^{ie\Lambda(x)}e^{-ie\Lambda(x)}\phi(x) \rightarrow |\phi(x)|^2 \)

Any expressions of power of \( |\phi(x)|^2 \) are gauge invariant.

(b) The covariant derivative

\[ D_\mu \phi(x) = (\partial_\mu + ieA_\mu)\phi(x) \rightarrow (\partial_\mu - ie\partial_\mu\Lambda(x) + ieA_\mu + ie\partial_\mu\Lambda(x))\phi(x) = D_\mu \phi(x) \]

A similar treatment can be done for the complex conjugate, hence the combination is gauge invariant.

(c) \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) transforms to

\[ \partial_\mu A_\nu - \partial_\nu \partial_\mu\Lambda(x) - \partial_\nu A_\mu + \partial_\mu \partial_\nu\Lambda(x) = F_{\mu\nu} \]

Here we use the fact that the order of the partial derivatives can be swapped. Hence \( F_{\mu\nu} F_{\mu\nu} \) is gauge invariant.

Therefore we can conclude that the Lagrangian density is invariant under the local U(1) gauge transformation.

2. The classical equations of motion can be derived by using the Euler-Lagrange equations for \( \phi \) and \( A \)

\[ \partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \phi)} - \frac{\delta \mathcal{L}}{\delta \phi} = 0 \rightarrow (\partial_\mu \partial^\mu + e^2 A^\mu A_\mu + m^2 + \frac{\lambda}{12} |\phi|^2)\phi^* + ie(\partial_\mu A^\mu)\phi^* + ieA^\mu \partial_\mu \phi^* = 0 \]

\[ \partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \phi^*)} - \frac{\delta \mathcal{L}}{\delta \phi^*} = 0 \rightarrow (\partial_\mu \partial^\mu + e^2 A^\mu A_\mu + m^2 + \frac{\lambda}{12} |\phi|^2)\phi - ie(\partial_\mu A^\mu)\phi - ieA^\mu \partial_\mu \phi = 0 \]

\[ \partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu A_\nu)} - \delta \frac{\delta \mathcal{L}}{\delta A_{\nu}} = 0 \rightarrow \partial_\mu F_{\mu\nu} - ie(\phi^* D^\nu \phi - \phi D^\nu \phi^*) = 0 \]

3. The Hamiltonian is related to the Lagrangian by the Legendre transformation. One needs to find the conjugate momenta for each of the fields, \( \frac{\delta \mathcal{L}}{\delta (\partial_\mu \phi)} \):

\[ \Pi = (D^0 \phi)^* \]
\[ \Pi^* = D^0 \phi \]
\[ \Pi^i = -F^{0i} \]

Note that the only non-zero components of \( \Pi^\nu \) are the spatial components, \( \Pi^i = E^i \).

This implies that the zero component of the vector potential is not a dynamic field. In stead it gives the constraint to the theory:

\[ \partial_i F^{0i} - ie(\phi^* D^0 \phi - \phi D^0 \phi^*) = 0 \rightarrow \nabla \cdot \vec{E} = \rho \]

which is the familiar Gauss law.
The Hamiltonian then is given by
\[ H = \Pi \partial_0 \phi + \Pi^* \partial_0 \phi^* + \Pi^i \partial_i A_i - \mathcal{L} \]
\[ = \Pi^* \Pi + (D_i \phi)^*(D^i \phi) + \frac{m^2}{2} \phi^* \phi + \frac{\lambda}{4!} (\phi^* \phi)^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]
Note that \( \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \) can be written in terms of the electric and magnetic fields as \( \frac{1}{2}(|\vec{E}|^2 + |\vec{B}|^2) \).

4. We go back to the Lagrangian and write it down in terms of \( \rho, \theta \).

\[ D_\mu \phi = (\partial_\mu \rho + i \rho (\partial_\mu \theta + e A_\mu)) e^{i \theta} \]

The lagrangian then can be written as
\[ \mathcal{L}(\rho, \theta, A^\nu) = \frac{1}{2} \partial_\mu \rho \partial_\mu \rho + \rho^2 (\partial_\mu \theta + e A_\mu)^2 - \frac{m^2}{2} \rho^2 - \frac{\lambda}{4!} \rho^4 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

The equations of motion then can be derived from the Euler-Lagrange equation
\[ \partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \rho)} - \frac{\delta \mathcal{L}}{\delta \rho} = 0 \rightarrow \partial_\mu \partial^\mu \rho + m^2 \rho + \frac{\lambda}{3!} \rho^3 - 2 \rho \rho_\mu (\partial_\mu \theta + e A_\mu)^2 = 0 \]
\[ \partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \theta)} - \frac{\delta \mathcal{L}}{\delta \theta} = 0 \rightarrow 2 \rho^2 (\partial_\mu \theta + e \partial_\mu A_\mu) + 2 \partial_\mu \rho (\partial_\mu \theta + e A_\mu) = 0 \]
\[ \partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu A_\nu)} - \frac{\delta \mathcal{L}}{\delta A_\nu} = 0 \rightarrow \partial_\mu F^{\mu \nu} + \rho^2 (2 e \partial^\mu \theta + e^2 A^\nu) = 0 \]

In the London gauge, \( \theta = 0 \) The Lagrangian then reduces to
\[ \mathcal{L}(\rho, A^\nu) = \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + \rho^2 e^2 A_\mu^2 - m^2 \rho^2 - \frac{\lambda}{4!} \rho^4 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]

5. In the unitary gauge one can write down the equations of motion as
\[ \partial_\mu \partial^\mu \rho + m^2 \rho + \frac{\lambda}{3!} \rho^3 - 2 e \rho^2 A_\mu^2 = 0 \]
\[ \rho^2 e \partial_\mu A_\mu + 2 (\partial_\mu \rho) e A_\mu = 0 \]
\[ \partial_\mu F^{\mu \nu} + e^2 \rho^2 A^\nu = 0 \] (5)

If we set \( \rho = \bar{\rho} = \text{constant} \), \( \partial^2 \rho = 0 \). From the first equation of motion, we know that \( \bar{\rho} \) has to satisfy
\[ \frac{\lambda}{3!} \bar{\rho}^2 = 2 e^2 A_\mu^2 - m^2 \]
Solving this yields
\[ \bar{\rho}^2 = -6 \frac{m^2}{\lambda} \]

Now one can fix \( \rho \) in the Lagrangian to obtain the effective Lagrangian
\[ \mathcal{L}(A_\mu) = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} (2 \bar{\rho} e^2) A_\mu^2 + \text{constant} \]
The equation of motion is

\[ \partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A_\mu - m_\Lambda^2 A^\nu = 0 \]

where \( m_\Lambda^2 = 2\bar{\rho} e^2 \). The second equation from 5 implies that \( \partial_\mu A^\mu = 0 \) as a constraint. We are left with

\[ \partial_\mu \partial^\mu A^\nu - m_\Lambda^2 A^\nu = 0 \]

which is the Klein-Gordon equation of the gauge field with mass \( m_\Lambda^2 = 2\bar{\rho} e^2 \).