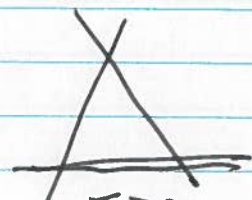
Other Dimer States

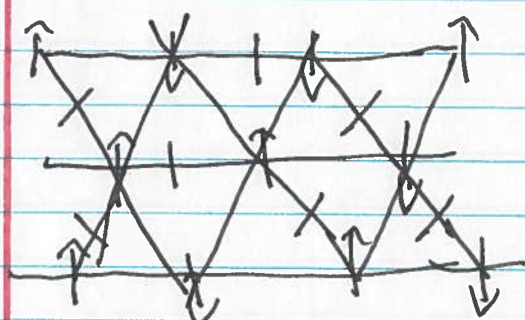
Consider a <u>frustrated</u> Ising model in a transverse field, e.g. an Ising Antiferromagnet on a triangular lattice



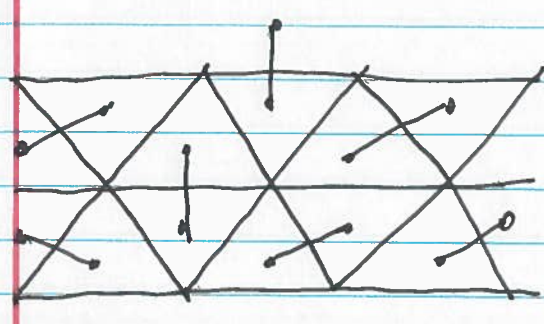
$$H = J \sum_{\langle r, r' \rangle} \sigma_3(r)\, \sigma_3(r') - h_T \sum_r \sigma_1(r)$$

$J > 0$

<u>extensive classical degeneracy</u> $(J \gg h_T)$



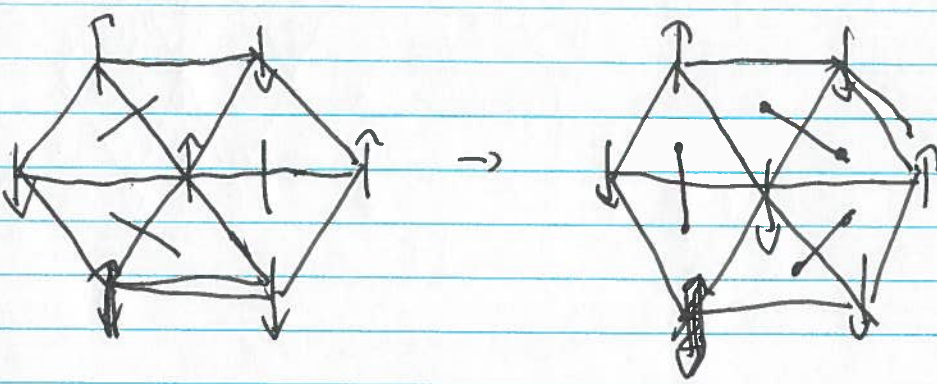
every triangle has an unhappy (broken) bond. The dual of the triangular lattice is the hexagonal lattice. Draw a <u>dimer</u> connecting neighboring sites of the hexagonal lattice that cross each broken bond. $\Rightarrow$ the # of dimer coverings of the hexagonal lattice counts the degeneracy.



$\left(\frac{S(0)}{k_B} = 0.323\ldots \right)$

The transverse field induces spin flips ⇒

⇒ connects degenerate states



"flippable config."

⇒ use an effective Hamiltonian on the degenerate subspace

⇒ <u>Quantum Dimer Model</u> (Rokhsar, Kivelson 1988)

(square lattice)

$$H_{QDM} = -J\left(|=\rangle\langle ||| + h.c.\right) + V\left(|=\rangle\langle =| + |||\rangle\langle |||\right)$$

flips plaquettes w/ parallel dimers

counts plaquettes with parallel dimers

<u>Special case:</u> $J = V$

More generally, let C be a dimer configuration. Assume that $\langle C | C' \rangle = \delta_{C,C'}$

(i.e. are orthogonal and linearly independent)

(this is not exactly correct for SU(2) singlets)

$\Rightarrow |\{C\}\rangle$ is a basis in this space of states

If $J = V \Rightarrow$ sum of (non commuting) projection operators $\Rightarrow H$ is non-negative

$\Rightarrow$ it has "zero modes", i.e. states s.t.

$$H |\Psi\rangle = 0$$

Example $\quad |\,||\,\rangle = |1\rangle, \quad |=\rangle = |0\rangle$

$$H = -J\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) + V\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

If $J = V \Rightarrow |\pm\rangle = |0\rangle \pm |1\rangle$

$$H = J |-\rangle\langle -|$$

$\Rightarrow$ G. state $|+\rangle$ / $H|+\rangle = 0$

In general (short-range RVB)

$$|\Psi\rangle = \sum_{\{C\}} |\{C\}\rangle \quad \text{equal amplitude superposition}$$

If $V \gg J \Rightarrow$ $|||\rangle\langle|||+|=\rangle\langle=|$ dominates

$\Rightarrow$ if $V<0 \Rightarrow$ columnar states

if $V \gg J$ $(V>0)$ $\Rightarrow$ staggered states.

In ~~generally~~ general the phase diagram depends on the lattices

* If the lattice is bipartite $\Rightarrow$ ordered phases and a critical point

* If the lattice is not bipartite $\Rightarrow$ we also have topological phases.

(I) Bipartite lattices at the KR point $(J=V)$

$$\Rightarrow |\Psi\rangle = \sum_{\{C\}} |C\rangle$$

$\uparrow$ dimer coverings

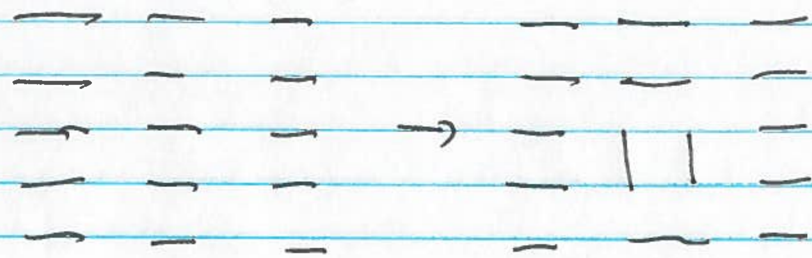
Let $\hat{A}$ be an operator diagonal in the dimer basis. $\Rightarrow$ $\hat{A}|C\rangle = A(C)|C\rangle$

$$\Rightarrow \langle\Psi|\hat{A}|\Psi\rangle = \sum_{\{C\}} A(C)$$

$\Rightarrow$ same as the exp. value of A in classical dimer model

Dimer Configurations and Loops

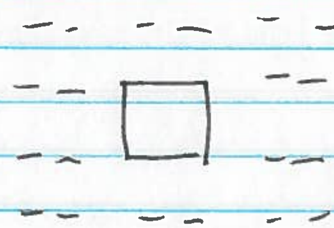
Consider two dimer coverings e.g. columnar state and a state with one flipped plaquette



(A) (B)

Construct the <u>transition graph</u> which shows which plaquettes have been flipped

(A) → B



loop

Each flipped plaquette is represented by a loop ⇒ the # of closed loop coverings of the lattice (not closed packed!) yields an equiv. representation ⇒ $|RK\rangle \Longleftrightarrow$ sum over <u>all</u> loops

Dimer density correlator = probability of finding two ~~p~~parallel dimers at distance R

Square lattice: $G(R) \sim \frac{1}{R^2}$
(Fisher, Stephenson)

Correlator of two monomers $C(R)$
(i.e. no dimers ~~at~~ touching these sites)

$C(R) \sim \frac{1}{R^\eta}$ $\eta = \frac{1}{2}$

(Problem is closely related to the Onsager problem)

This is true if the lattice is bipartite

Also, Rokhsar and Kivelson did a variational calculation to compute an excited collective mode

$|\vec{k}\rangle = \Psi_{\vec{k}} |G.S.\rangle = \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}\, \psi(\vec{R})\, |G.S.\rangle$

If $\Psi(\vec{R})$ is the dimer density op. $\Rightarrow$

$|\vec{k}\rangle$ is a "phonon"

They used Feynman's S.M.A. and found ("resonon")

$E(\vec{k}) = \text{const}\, |\vec{k}|^2 + \ldots$ $(\vec{k} \sim (\pi,\pi)) \Rightarrow z = 2$

Non-Bipartite Lattices $\Rightarrow$ Topological Phases

Consider now a QDM on a non-bipartite lattice (e.g. triangular)

$\Rightarrow$ we still get a sum over dimer coverings but on a triangular lattice

Moessner, Sondhi ($\sim$2001) found that in this case the SR-RVB state is not critical and has a finite correlation length $\xi \sim a$

$\Rightarrow$ "dimer liquid"

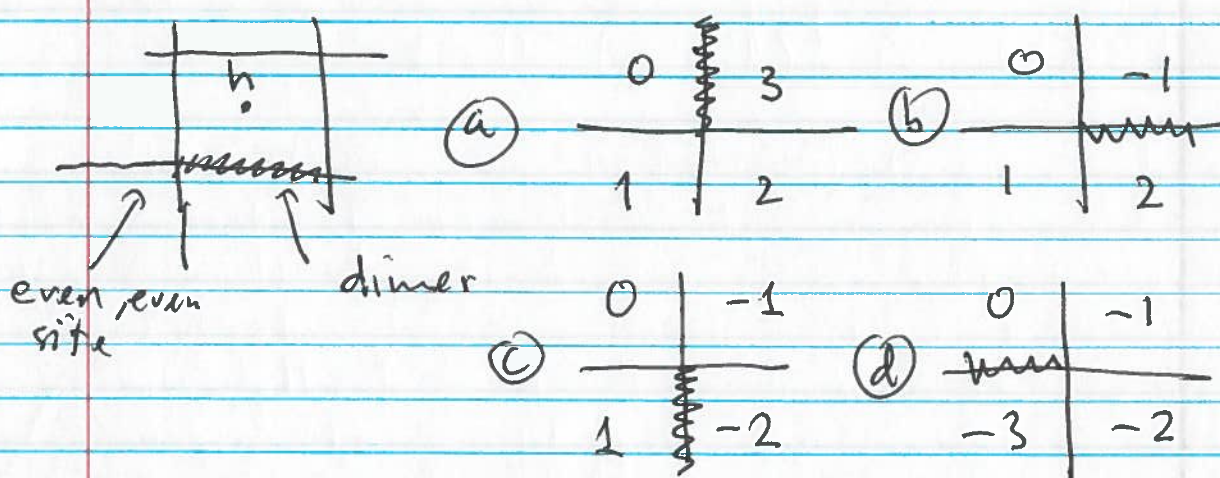
$\Rightarrow$ $|\Psi\rangle$ is now equiv. to a sum over all loop configurations with the same amplitude

$\Rightarrow$ Same as the Kitaev state for the Toric Code (a.k.a. the deconfined phase of the $\mathbb{Z}_2$ gauge theory) which we saw is a topological phase

Quantum Dimers @ Criticality

(Moessner, Sondhi & EF ~ 2001
Ardonne, Fendley, & EF ~ 2004)
(also C. Henley ~ 1996)

Consider a square lattice and label the dimer configs. Eq as "heights" on the dual lattice $h \in \mathbb{Z}$



heights are defined up to an uniform shift by 4 and wind around the sites of the direct lattice.

Average heights

(a) $\Leftrightarrow$ $h = 3/2$, (b) $h = \frac{1}{2}$

(c) $h = -1/2$, (d) $h = -3/2$

We will construct a Hamiltonian for

(1) the field $h \in \mathbb{Z}$

(2) $h \equiv h + 4n$ defines an equivalence.

$\Rightarrow$ h will be represented by a scalar field (a phase field) with compactification radius $R = 4$

(3) All operators must be invariant under these shifts

(4) For a special value of a parameter the equal-time correlators must be the same as in the dimer model $\Leftrightarrow$ RK point

Let's rescale h by $\pi/2$ and define

$\varphi = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$\varphi = 0, \pi \Rightarrow$ columnar states (horizontal dimers)

$\frac{\pi}{2}, \frac{3\pi}{2}$ (vertical dimers)

$\Rightarrow$ 4 columnar states

Classical Dimers and Critical Phenomena

$$S_{2D} = \int d^2x \, \frac{K}{2} (\vec{\nabla} \varphi)^2$$

(see Nienhuis 1987, Henley & Kondev 1996)

& Cauchy-Riemann

$$\partial_\mu \varphi = \epsilon_{\mu\nu} \partial_\nu \vartheta$$

$$V_n(x) = e^{in\varphi(x)}$$ ("electric") (charge)

$$\tilde{V}_m(x) = e^{im\vartheta(x)}$$ ("magnetic") (vortex)

dimer densities

$$n_x = \frac{1}{4} + \frac{1}{2\pi}(-1)^{x+y} \partial_y \varphi + \frac{1}{2}\left[(-1)^x e^{i\varphi} + c.c.\right]$$

$$n_y = \frac{1}{4} + \frac{1}{2\pi}(-1)^{x+y+1} \partial_x \varphi + \frac{1}{2}\left[(-1)^y e^{i\varphi} + c.c.\right]$$

$\Rightarrow V_1(x) \equiv e^{i\varphi(\vec{x})}$ takes the values $1, i, -1, -i$

$\Rightarrow$ columnar order parameter $O_c(x)$

$V_2(x) \equiv e^{2i\varphi(x)}$ takes values $1, -1, 1, -1$

⊗ orientational order $N(x)$ ("nematic")