L12 9/29/2022

$$\Rightarrow \Psi[\varphi] = \frac{1}{\sqrt{Z_0}} e^{-\int d^2x \, \frac{\kappa}{2} (\vec{\nabla}\varphi)^2} \, !$$

Norm

$$Z_0 = \int \mathcal{D}\varphi \; e^{-\int d^2x \; \kappa (\vec{\nabla}\varphi)^2}$$

same as the P.F. of a compactified boson in 2D!

$$K \equiv \cancel{2\kappa}\; 2\kappa$$

$\Rightarrow$ All ops. which are <u>diagonal</u> in the φ rep. have equal time correlator identical to the classical correlators!

Recall $\vec{E} = \vec{\nabla} \times \varphi$

$$\Rightarrow \Psi[\vec{E}] = \frac{1}{\sqrt{Z_0}} e^{-\int d^2x \, \frac{\kappa}{2} \vec{E}^2(x)} \prod_x \delta(\vec{\nabla}\cdot\vec{E})$$

and

$$\langle \Psi_{GS} | O_{n_1}(\vec{x}_1) \cdots O_{n_N}(\vec{x}_N) | \Psi_{GS} \rangle \equiv$$

$$= \langle O_{n_1}(x_1) \ldots O_{n_N}(x_N) \rangle^{\text{classical}}_{K = 2\kappa}$$

Since we know that for classical dimers (square lattice) $K_{free} = \frac{1}{4\pi}$

$\Rightarrow$ $\mathcal{K}_{free} = \frac{1}{8\pi}$

And the scaling dimensions are the <u>same</u> in both theories $\Rightarrow$

the scaling dimension of the operator $\mathcal{O}_n[\varphi]$ in the QLM is $\Delta_n = \frac{1}{8\pi K}$

<u>Vortex Operators</u> (holes in the QDM)

$$\tilde{\mathcal{O}}_m(\vec{x}) = e^{i\int d^2z\, \alpha(\vec{z})\, \Pi(\vec{z})}$$

$$\alpha(\vec{z}) = m \arg(\vec{z}-\vec{x}) \qquad (0 \leq \arg(z-x) \leq 2\pi)$$

and $e^{i\int d^2z\, \alpha(z)\Pi(z)} \, |\{\varphi\}\rangle = |\{\varphi(x) - \alpha(x)\}\rangle$
shift

This is a <u>singular gauge transf.</u>

$\Leftrightarrow$ to coupling φ to a vector field $\vec{A}(z)$ /

$$\oint_\gamma d\vec{z} \cdot \vec{A}(z) = 2\pi m \quad \text{if } x \text{ is } \underline{\text{inside}}\ \gamma$$

$$\Rightarrow \quad \Psi_m(\vec{x}) = \langle \{\varphi\} | \tilde{O}_m(\vec{x}) | \Psi_0 \rangle$$

$\uparrow$ vortex

$$= \frac{1}{Z_0} e^{-\frac{\kappa}{2}\int d^2z \left(\vec{\nabla}\varphi - \vec{A}\right)^2}$$

Same with many vortices /

$$\varepsilon_{ij}\partial_i A_j = 2\pi \sum_{l=1}^{M} m_l \, \delta^2(z-x)$$

Scaling dimension $\tilde{\Delta}_m = 2\pi\kappa m^2 \Rightarrow$ Pairs of holes (diag. dimer) $\tilde{\Delta}_2 = 1$

* How about the time dependence?

~~$G(\vec{x}-\vec{x}', t-t')$~~ (imaginary time)

$$G(\vec{x}-\vec{x}', \tau-\tau') = \langle \varphi(\vec{x}, \tau)\, \varphi(\vec{x}', \tau') \rangle$$

$$= \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \int \frac{d^2q}{(2\pi)^2} \frac{e^{i\omega(\tau-\tau') - \vec{q}\cdot(\vec{x}-\vec{x}')}}{\omega^2 + \kappa^2(\vec{q}^{\,2})^2}$$

pole in real time $\quad \omega(\vec{q}) = \kappa \vec{q}^{\,2} \qquad (z=2)$

short distance cutoff

$$G_{reg}(\vec{x}, \tau) \equiv G(\vec{x}, \tau) - G(a, 0)$$

$$= -\frac{1}{8\pi\kappa}\left[\ln\left(\frac{|\vec{x}|^2}{a^2}\right) + \Gamma\left(0, \frac{|\vec{x}|^2}{4\kappa|\tau|}\right)\right]$$

$$\Gamma(0,z) = \int_z^\infty \frac{ds}{s}\, e^{-s} \quad \text{(incomplete } \Gamma\text{-function)}$$

$$\Rightarrow G_{reg}(\vec{x},\tau) = \begin{cases} -\dfrac{1}{4\pi K} \ln\left(\dfrac{|\vec{x}|}{a}\right) & |\tau| \to 0 \\ -\dfrac{1}{8\pi K} \ln\left(\dfrac{4K|\tau|}{a^2\gamma}\right) & |\vec{x}| \to 0 \end{cases}$$

$\gamma = C = 0.577\ldots$ Euler constant

$$\Rightarrow \langle \mathcal{O}_n(x,\tau)^\dagger \, \mathcal{O}_n(x',\tau') \rangle = e^{n^2 G_{reg}(|x-x'|,\, \tau-\tau')}$$

$$\Rightarrow |\tau-\tau'| \to 0, \quad \langle \mathcal{O}_n(x,\tau)^\dagger \, \mathcal{O}_n(x',\tau') \rangle = \left(\frac{a}{|x-x'|}\right)^{n^2/4\pi K}$$

$$|x-x'| \to 0, \quad \langle \mathcal{O}_n(x,\tau)^\dagger \, \mathcal{O}_n(x,\tau) \rangle = \left(\frac{a^2\gamma}{4K|\tau-\tau'|}\right)^{\frac{n^2}{8\pi K}}$$

$\Rightarrow \quad z = 2$

<u>Relevance and Irrelevance:</u>

Since $z=2 \Rightarrow$ effective Euclidean dimension is $D = z + d = 4 \quad (d=2)$

$\Rightarrow \Delta > 4$ irrelevant op.

$\Delta < 4$ relevant op.

$\Delta = 4$ marginal op.

$\Rightarrow$ $(\vec{\nabla}\varphi)^2$ has $\Delta = 2 \Rightarrow$ relevant

$g_4 (\nabla\varphi)^4$ has $\Delta = 4$, marginal?

a one-loop computation shows that

$$\beta(g_4) = a \frac{\partial g_4}{\partial a} = -c g_4^2 + \cdots$$

$\Rightarrow$ marginally irrelevant. (log. corrections)

Other lattices?

Honeycomb Lattice

we can write an op.

$$g_3 (\partial_x\varphi)\left((\partial_x\varphi)^2 - 3(\partial_y\varphi)^2\right)$$

invariant under $\pi/3$ rotations and $\varphi \to -\varphi$

scaling dimension $\Delta_3 = 3 \Rightarrow$ relevant

$\Rightarrow$ g_3 grows at low energies

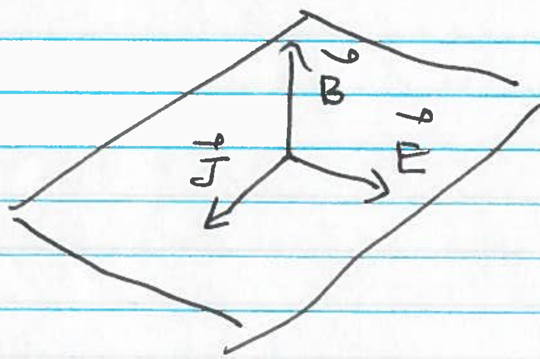
$\Rightarrow$ expect $\xi < \infty$ and a 1st order transition

Topology and the Quantum Hall Effect

(see chapter 12, EF Field Theories of CM Systems)

Classical Hall Effect

Electrons moving freely on a surface with a $\perp$ $\vec{B}$ and an in-plane $\vec{E}$ have a current $\vec{J}\cdot\vec{B} = \vec{J}\cdot\vec{E} = 0$



why: Lorentz force

$$\vec{F} = \frac{e}{c}\,\vec{v}\times\vec{B} - e\vec{E}$$

$$\Rightarrow \vec{F} = 0$$

drift current $\frac{1}{c}\,\vec{v}\times\vec{B} = \vec{E}$

$$\Rightarrow |\vec{v}| = \frac{|\vec{E}|c}{|\vec{B}|}, \quad \vec{J} = -e\vec{v}$$

Conversely, an injected current generates a Hall voltage in the $\perp$ direction of the plane

Landau Levels (Landau 1930)

Landau solved the quantum mechanics of a particle in a magnetic field

Spectrum: Landau levels

$$E_n = \hbar\omega_c\left(m + \tfrac{1}{2}\right) \qquad m = 0, 1, 2 \ldots$$

$$\omega_c = \text{cyclotron frequency} = \frac{eB}{Mc}$$

mass

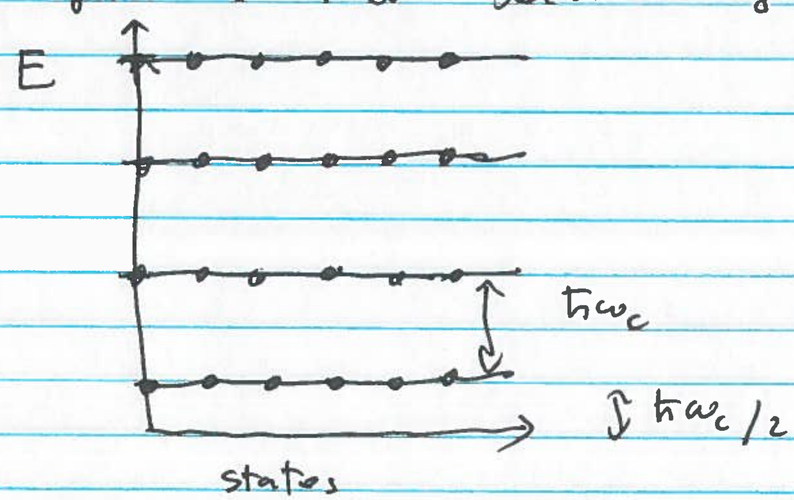
Each level has a degeneracy = # of flux quanta

$$\text{Flux} = BL^2 = \Phi = N_\phi \, \phi_0$$

$$\text{flux quantum} = \phi_0 = \frac{hc}{e}$$

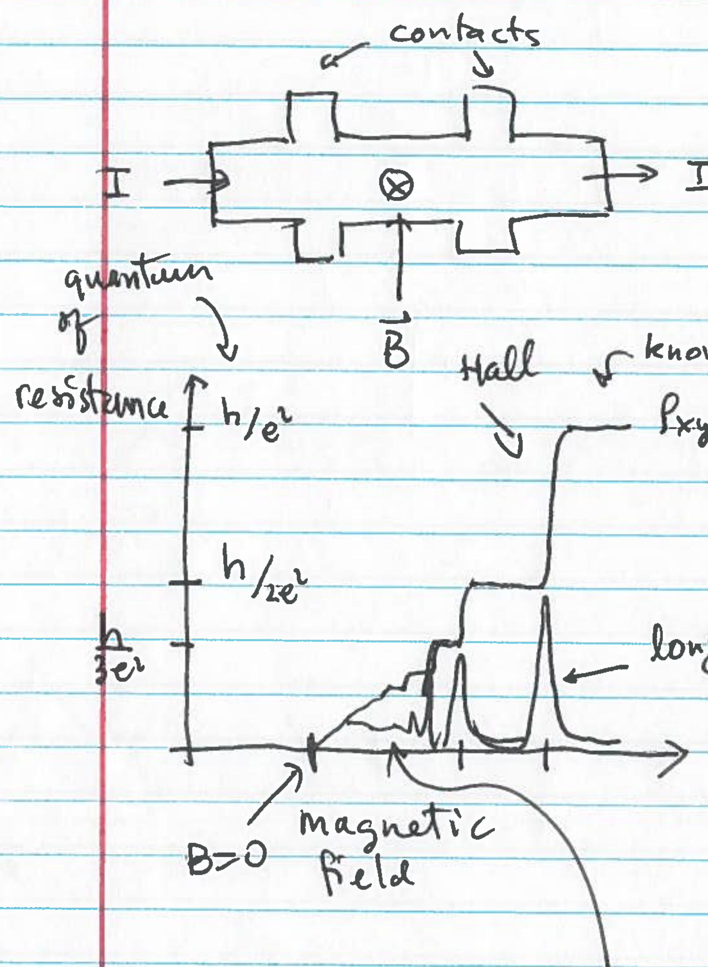
$\Rightarrow$ degeneracy $= N_\phi$ (each level!)

degenerate states labeled by $n = 0, 1, \ldots, N_\phi - 1$



* Remarkable discovery by Klaus von Klitzing (1980)

He was measuring the Hall resistance in a MOSFET (Metal Oxide Field Effect Transistor)



$$\rho_{ij} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}$$

resistivity tensor

known to 10^{-9}

$$\rho_{xy} = -\rho_{yx}$$

conductivity tensor

$$\sigma_{ij} = \frac{1}{\det \rho} \begin{pmatrix} \rho_{yy} & -\rho_{xy} \\ -\rho_{yx} & \rho_{xx} \end{pmatrix}$$

$$\det \rho = \rho_{xx}\rho_{yy} + \rho_{xy}^2$$

Shubnikov - de Haas Oscillations

ρ_{xx} (and ρ_{yy}) $\to 0$ when ρ_{xy} is at a plateau

Non-dissipative ~~current~~ Hall current @ plateau

dissipation only exists in transitions between plateaus