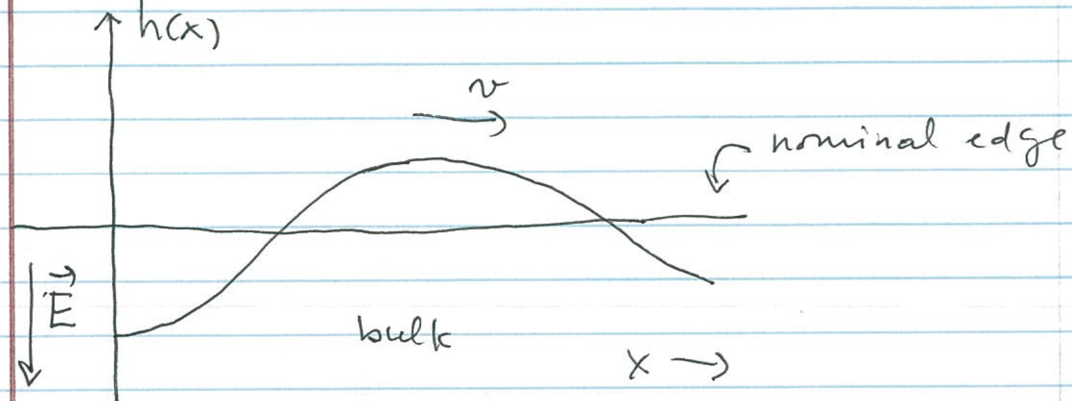
Edge States in FQH fluids

In Lecture 21 (11/1/22) we discussed edge states in non-interacting topological systems such as the IQH free fermion model. Also in Lecture 24 (11/15) we discussed that a Chern-Simons theory on a disk ~~has~~ requires edge states to restore gauge invariance. We also showed that the effective action for the edge state is

$$\mathcal{L} = \frac{k}{4\pi} \varepsilon_{\mu\nu\lambda} b^\mu \partial^\nu b^\lambda$$

We will now look at this problem using a hydrodynamic picture (Wen ~ '90). Let $n_0 = \frac{\nu}{2\pi l_0^2}$ be the 2D uniform particle density and $l_0 = \sqrt{\frac{\hbar c}{eB}}$ be the magnetic length. The bulk 2D liquid is incompressible (due to the large energy gap) $\Rightarrow$ the only low energy excitations are the fluctuations of the ~~shape~~ shape at the boundary of the 2D liquid



Near the edge there is a drift current ~~flowing~~

$$\vec{J} = \sigma_{xy}\, \hat{z} \times \vec{E} \quad \left(\text{with a velocity } v = \frac{|\vec{E}|\, c}{B}\right)$$

flowing along the edge, where $\sigma_{xy} = \frac{\nu}{2\pi} \frac{e^2}{\hbar}$

The local ~~xy~~ edge displacement is $h(x)$

an the 1D density (straight edge) $\rho(x) = n_0 h(x)$

The density is a chiral wave that obeys

$$\partial_t \rho(x,t) - v\, \partial_x \rho(x,t) = 0 \qquad \left(v \text{ is the drift velocity}\right)$$

In the limit in which $|h| \ll R$ (radius of the ~~edge~~ 2DEG droplet) and with $\vec{E}$ uniform the total electrostatic energy H stored at the edge is

$$H = \int dx \, \frac{1}{2} e h \rho(x) E = \int dx \, \frac{\pi \hbar}{\nu} v \rho^2(x)$$

$L = 2\pi R$ is the length (perimeter) of the edge

since the edge is a closed boundary

$$\Rightarrow \rho(x) = \rho(x+L)$$

Fourier transform

$$\rho(x) = \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{+\infty} e^{i \frac{2\pi n}{L} x} \rho_n$$

$$\rho_n = \frac{1}{\sqrt{L}} \int_0^L dx \, e^{-i \frac{2\pi n x}{L}} \rho(x)$$

$$\Rightarrow H = \frac{\pi}{\nu} \hbar v \sum_{n=-\infty}^{+\infty} \rho_n \rho_{-n} = \frac{2\pi}{\nu} \hbar v \sum_{k>0} \rho_k \rho_{-k}$$

$k_n = \frac{2\pi n}{L}$ momentum

The classical eq. of motion of the Fourier modes is

$$\partial_t \rho_k = i v k \rho_k$$

In a Hamiltonian system $H(q, p)$

$$\dot{q} = \partial_p H \quad , \quad \dot{p} = -\partial_q H$$

$\Rightarrow$ we can identify a generalised coordinate

$$Q_k \equiv \rho_k \quad \text{and} \quad P_k = -i \frac{2\pi}{\nu k} \rho_{-k}$$

such that

$$\dot{Q}_k = \frac{\partial H}{\partial P_k} ; \quad \dot{P}_k = -\frac{\partial H}{\partial Q_k}$$

$$\Rightarrow \quad \dot{\rho}_k = i v k \rho_k \, , \, \dot{\rho}_{-k} = -i v k \rho_{-k}$$

$$\Rightarrow H = -i v \sum_{k>0} k \, Q_k P_k$$

<u>Quantization</u>: Q_k, P_k become operators acting on a Hilbert space and obey canonical commutation relations

$$[Q_k, P_{k'}] = i\hbar \, \delta_{k,k'} \qquad \left(k = \frac{2\pi n}{L}\right)$$

$$\Rightarrow [\rho_k, \rho_{k'}] = \frac{\nu}{2\pi} k \, \delta_{k+k',0}$$

This is known as a $U(1)$ Kac-Moody algebra at some integer level (generally $\neq 1$)

H is now an operator ~~which~~ which generates the time evolution of the operators (Heisenberg picture)

$\Rightarrow$ $[H, \rho_k] = \hbar v k \rho_k$

How do we describe adding an electron at the edge? Let $\psi_e^+(x)$ the operator that creates an electron at location x along the edge. Demand that

$$[\rho(x), \psi_e^+(x')] = \delta(x-x')\, \psi_e^+(x') \qquad \text{I}$$

Let us represent the local density $\rho(x)$ in terms of a scalar field $\phi(x)$ / $\rho(x) = \frac{1}{2\pi}\partial_x \phi(x)$

The total charge Q at the edge must vanish

$$Q = \int_0^L dx\, \rho(x) = 0 \Rightarrow \phi(x+L) = \phi(x) \quad \text{(PBC's)}$$

$\Rightarrow$ The operator that satisfies the commutation relation I is

$$\psi_e^+(x) \sim e^{\frac{i}{\nu}\phi(x)}$$

The Fourier transform of $\phi(x)$ is ϕ_k

and $Q_k = i \frac{k}{2\pi} \phi_k$, $P_k = -\frac{1}{\nu} \phi_k$

$\Rightarrow [\phi_k, \phi_{-k'}] = i\nu \, \delta_{k,k'}$

and $\dot{\phi}_k = i v k \phi_k$

$$\Rightarrow H = \int dx \frac{v}{4\pi\nu} (\partial_x \phi)^2 \qquad \nu = 1/m$$

and $$\mathcal{L} = \frac{m}{4\pi} \left[\partial_t \phi \, \partial_x \phi - v(\partial_x \phi)^2\right]$$

which is what we obtained from CS. theory!

$$\Rightarrow \psi_e(x) \psi_e(x') = e^{i\pi/\nu} \psi_e(x') \psi_e(x)$$

since $\nu = \frac{1}{m}$ and m is odd $\Rightarrow \{\psi_e(x), \psi_e(x')\} = 0$

Interactions:

$$H_{int} = \frac{1}{2} \int dx \int dx' \, \rho(x) \, V(x-x') \, \rho(x')$$

$$= \frac{1}{8\pi^2} \int dx \int dx' \, \partial_x \phi(x) \, V(x-x') \, \partial_{x'} \phi(x')$$

For short range interactions with forward scattering coupling constant g we see that they only renormalize the ~~velocity~~ velocity $v_{eff} = v + \frac{g}{2\pi}$

$\Rightarrow$ Edge modes with $\omega(k) = k v_{eff}$

~~Coulomb~~ Coulomb: $V(x-x') = \frac{e^2}{|x-x'|}$

$\Rightarrow$ in Fourier space $V(k) = -\frac{e^2}{2\pi} k \ln(|k| \ell_0)$

$\Rightarrow$ the edge of a FQH fluid is a chiral Luttinger liquid

Propagator $\langle T \phi(x,t) \phi(0,0) \rangle = -\nu \ln\left(\frac{x - vt + i\varepsilon}{a_0}\right)$

$$\Rightarrow G_F(x,t) = \langle T \psi_e^\dagger(x,t) \psi_e(0,0) \rangle$$

$$= e^{\frac{1}{\nu^2} \langle T \phi(x,t) \phi(0,0) \rangle}$$

$$\propto \frac{\text{const}}{(x-vt)^{1/\nu}}$$ (we omitted a factor $e^{ik_F x}$)

$\Rightarrow G_F(x,t) = \frac{\text{const}}{(x - vt + i\varepsilon)^m}$ which is odd for m odd.

$G_F(x,t) = -G_F(-x,-t)$

$\Rightarrow$ we do not have a simple pole (only for $m = \frac{1}{\nu} = 1$!)

Suppose that we now <u>add</u> (or <u>remove</u>) a single electron from the edge $\Rightarrow$

$$n_e = \int_0^L dx\, \rho(x) = \frac{1}{2\pi}\left(\phi(L) - \phi(0)\right)$$

$\Rightarrow \phi(x+L) = \phi(x) + 2\pi n_e \Rightarrow$ twisted BC's.

We saw before that a quasihole in the bulk amounts to an extra charge $\frac{e}{m}$ added at the edge

$$\Rightarrow \quad \psi_{qp}(x) \sim e^{i\phi(x)}$$

since

$$[\rho(x), \psi_{qp}^\dagger(x')] = \frac{1}{m}\,\delta(x-x')\,\psi_{qp}^\dagger(x')$$

qp propagator: $\langle T\, \psi_{qp}^\dagger(x,t)\, \psi_{qp}(0,0)\rangle =$

$$= e^{\langle T\, \phi(x,t)\, \phi(0,0)\rangle} = \frac{\text{const}}{(x - vt + i\varepsilon)^{1/m}}$$

$$\Rightarrow \langle T\, \psi_{qp}^\dagger(-x,-t)\, \psi_{qp}(0,0)\rangle = e^{\pm \frac{i\pi}{m}} \langle T\, \psi_{qp}^\dagger(x,t)\, \psi_{qp}(0,0)\rangle$$

This is actually a <u>compactified</u> ~~boson~~ chiral boson since it must obey $\phi(x) \equiv \phi(x) + 2\pi n$

$n \in \mathbb{Z}$

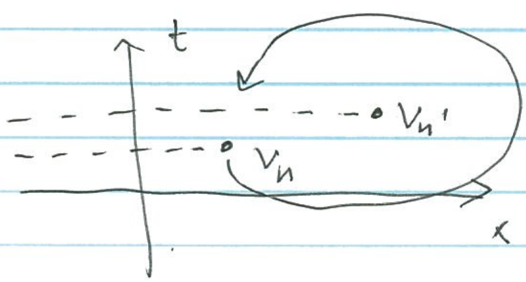
$\Rightarrow$ the admissible operators are

$$V_n(x) = e^{i n \phi(x)}$$

$$\Psi_e \equiv V_m$$

The operators V_n with $n < m$ are special since they are <u>local</u> w.r.t. Ψ_e (trivial braiding)

The edge picture of braiding is a monodromy



$\sim e^{i 2\pi n n'/m}$

$\Rightarrow V_n$ is single valued w.r.t. Ψ_e but not w.r.t. $V_{n'}$

Operators with $n = l m$ <u>are equivalent</u> to adding or removing l electrons (with their fluxes) $\Rightarrow$ no change in $\nu \Rightarrow$ only V_n with n defined mod m are physical.

$\{V_n\}$ $0 \leq n \leq m$ (m of them)

extended chiral algebra

<u>m</u> is identified with the <u>level</u> of the KM algebra

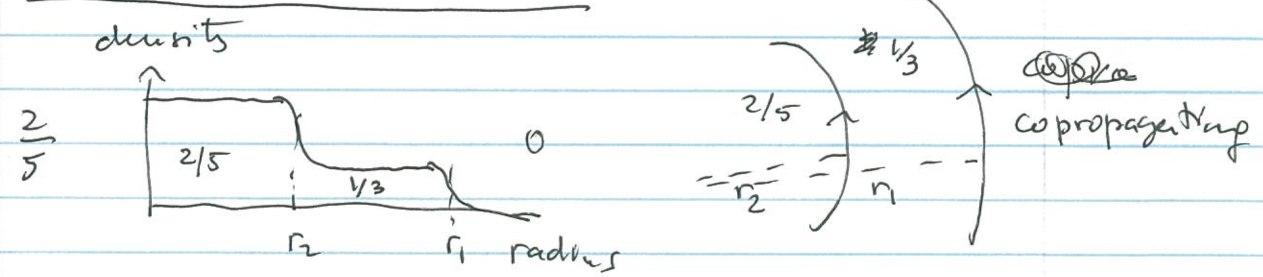
Scaling dimensions: $\Delta_n = \frac{n^2}{2m}$

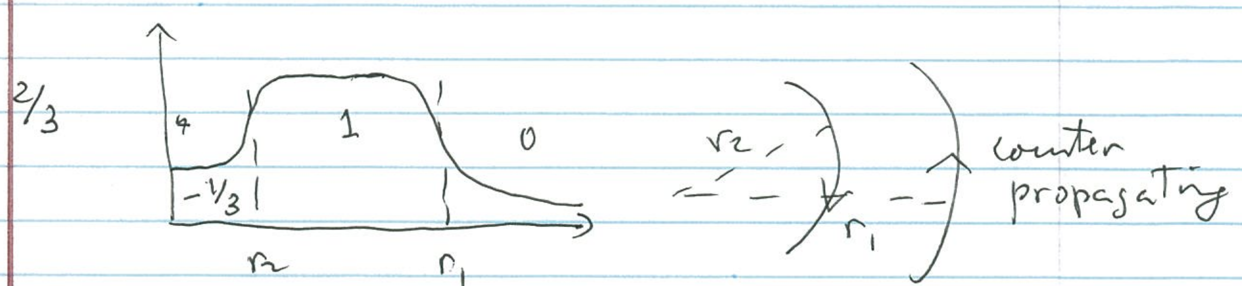
and a conformal spin $= \Delta_n$.

Electron $\Delta_e = \frac{m^2}{2m} = \frac{m}{2}$

Fund. Q.P. $\Delta_{qp} = \frac{1}{2m}$

<u>Edges of Jain states</u>





~~The general~~

Let $\rho_{I,k}$ $(I=1,2)$ be the modes

$$\rho_{I,k} = \frac{\nu_I}{2\pi l_0^2} h_I$$

$$\Rightarrow \left[\rho_{I,k}, \rho_{J,k'}\right] = \frac{\nu_I}{2\pi} \delta_{IJ} \, \delta_{k+k',0}$$

and $$H = 2\pi \sum_{I=1,2} \sum_{k>0} \frac{v_I}{\nu_I} \rho_{I,k} \, \rho_{I,-k}$$

$$\rho_I(x) = \partial_x \phi_I(x)$$

$$\psi_e = e^{\frac{i}{\nu_e} \phi_I(x)}$$

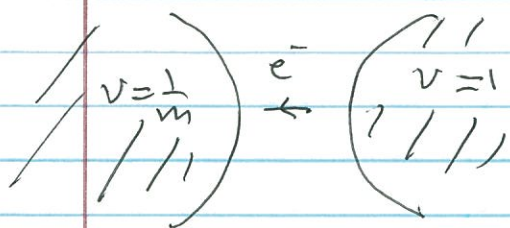
$$\langle T \psi_e(x) \psi_e^+(0) \rangle = \frac{e^{ik_I x}}{(x - v_I t + i\varepsilon)^{1/|\nu_I|}}$$

$$k_I = \frac{r_I}{2 l_0^2}$$

$$H_{\text{interedge}} = -2\pi \sum_{I,J} \sum_{k>0} V_{IJ} \, \rho_{I,k} \, \rho_{J,-k}$$

Tunneling Hamiltonians

Tunneling into an edge from a FL ($\nu=1$) state



$$H = H_{edge} + H_{FL} + H_{Tunnel}$$

$$H_{Tunnel} = \Gamma\, e^{i\omega_0 t}\, \psi^{+}_{e,edge}(0,t)\, \psi_{e,FL}(0,t) + h.c.$$

$\omega_0 = \frac{eV}{\hbar}$ "Josephson" frequency

$\Gamma =$ tunnel m.e.

$$G_e(x,t) = \frac{\text{const}}{(x - vt + i\varepsilon)^m} \qquad (\varepsilon \to 0^+)$$

Tunneling Density of States

$$N(\omega) = \operatorname{Im} \lim_{\varepsilon \to 0^+} \int_{-\infty}^{\infty} dt\; G_e(x,t)\, e^{i\omega t}$$

$$\sim \text{const} \times |\omega|^{m-1}$$

$$\int_{-\infty}^{+\infty} dt\, \frac{e^{i\omega t}}{(\beta \pm it)^{\alpha}} = \frac{2\pi}{\Gamma(\alpha)} (\pm\omega)^{\alpha-1} e^{\mp\beta\omega}\, \theta(\pm\omega)$$

Tunneling Current: use Fermi's Golden Rule

$$I_e(V) = \frac{2\pi e}{\hbar} |\Gamma|^2 \int_{-eV}^{0} dE \; N_{CLL}(E,T)\, N_{FL}(E+eV,T)$$

$$\sim V^m \qquad (V: \text{voltage})$$

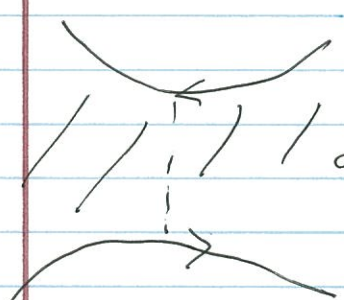
Diff. conductance

$$G_e(V,T) = \frac{dI_e}{dV} = \frac{2\pi}{\hbar} |\Gamma|^2 N_{FL}(0)\, \underbrace{N_{CLL}(V,T)}_{\sim V^{m-1}}$$

Note: it is not Ohmic!

$\Rightarrow$ tunneling is suppressed as $V \to 0$

constriction



$$H = H_R + H_L + \Gamma e^{i\omega_0^* t}\, \Psi^\dagger_{qpL}(0,t)\, \Psi_{qpR}(0,t)$$

$$\omega_0^* = g^* \frac{V}{\hbar} \qquad g^* = \frac{e}{m}$$

Q.P. current $\quad J_{qp} = i\Gamma \frac{e}{m} \left(\Psi^\dagger_{qpL}(0,t)\, \Psi_{qpR}(0,t) - h.c. \right)$

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Qp. DOS $N_{qp}(\omega) \sim |\omega|^{\frac{1}{m}-1}$

$$G_{qp}(V) = \frac{dI_{qp}}{dV} \propto V^{2\left(\frac{1}{m}-1\right)}$$

$$I_{qp}(V) \sim V^{\frac{2}{m}-1}$$